1.2 Reflections and Stretches

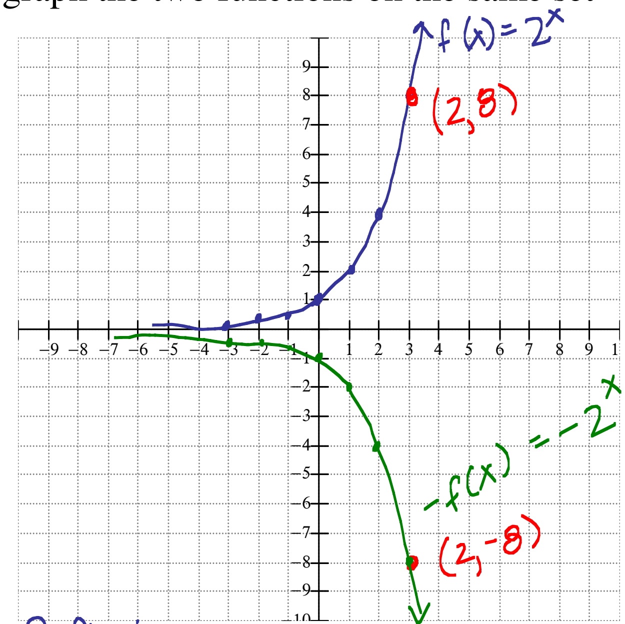
Ex. #1: Complete the table of values and graph the two functions on the same set of axes.

$-f(-3) = -2^{-3} = -\frac{1}{2^3} = -\frac{1}{8}$

x	f(x) = 2^x	-f(x) = -2^x
-3	1/8	-1/8
-2	1/4	-1/4
-1	1/2	-1/2
0	1	-1
1	$2^1 = 2$	-2
2	$2^2 = 4$	-4
3	$2^3 = 8$	-8

$f(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$



Describe how the two graphs are related: Reflection over the x-axis

In general, the graph of y = -f(x) is a *reflection* of the graph of y = f(x) in the x - axis. Mapping Notation: $(x, y) \rightarrow (x, -y)$

Ex. #2: Complete the table of values and graph the two functions on the same set of axes.

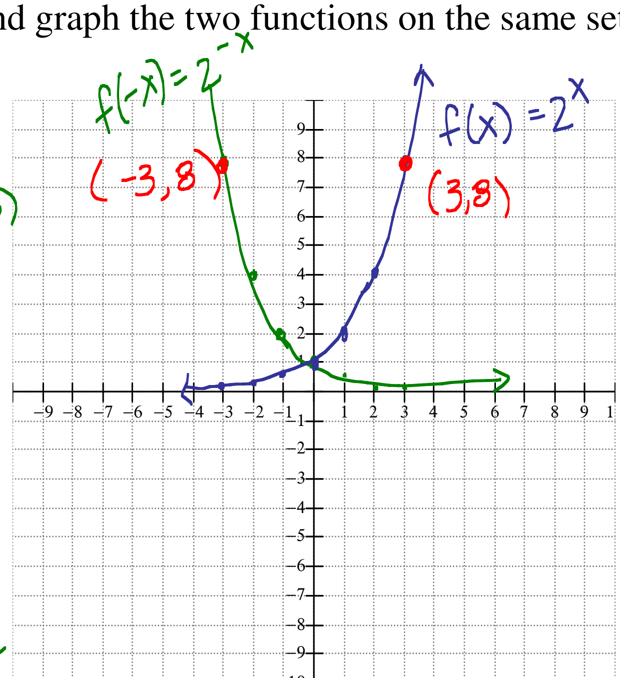
x	f(x) = 2^x	f(-x) = 2^{-x}
-3	1/8	8
-2	1/4	4
-1	1/2	2
0	1	1
1	2	1/2
2	4	1/4
3	8	1/8

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$f(-(-3)) = 2^{-(-3)} = 2^3 = 8$

$2^{-(-2)} = 2^2 = 4$

$2^{-(1)} = \frac{1}{2^1} = \frac{1}{2}$



Describe how the two graphs are related: Reflection over the y-axis

In general, the graph of y = f(-x) is a *reflection* of the graph of y = f(x) in the
__y__- axis. Mapping Notation: $(x, y) \rightarrow (\underline{-x}, \underline{y})$

Ex. #3: Sketch the graph of $y = |x|$, $y = 2|x|$, and $y = \frac{1}{3}|x|$ on the axes below.

$\boxed{y = |x|}$

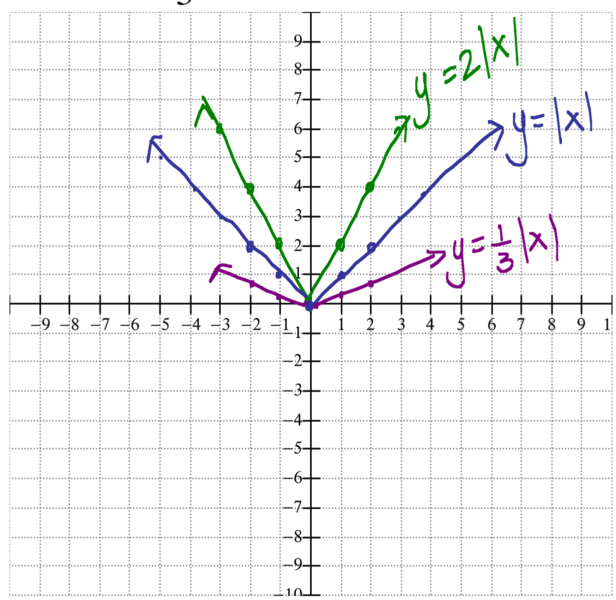
x	y
-2	2
-1	1
0	0
1	1
2	2

$\boxed{y = \boxed{2}|x|}$

x	y
-2	4
-1	2
0	0
1	2
2	4

$\boxed{y = \frac{1}{3}|x|}$

x	y
-2	2/3
-1	1/3
0	0
1	1/3
2	2/3



In general, for any function y =f(x), the graph of y = **a**f(x), where **a** is any real number results in a __Vertical stretch__. Mapping Notation: $(x, y) \rightarrow (\underline{x}, \underline{ay})$

Ex. #4: Graph the functions $y = \sqrt{x}$, $y = \sqrt{2x}$, and $y = \sqrt{\frac{1}{2}x}$ on the axes below.

$\boxed{y = \sqrt{x}}$

x	y
0	0
1	1
4	2
9	3

$\boxed{y = \sqrt{2x}}$

x	y
0	0
$\frac{1}{2}$	1
2	2

$y = \sqrt{2(0)}$
$y = \sqrt{0}$
$= 0$

$y = \sqrt{2(\frac{1}{2})}$
$= \sqrt{1}$
$= 1$

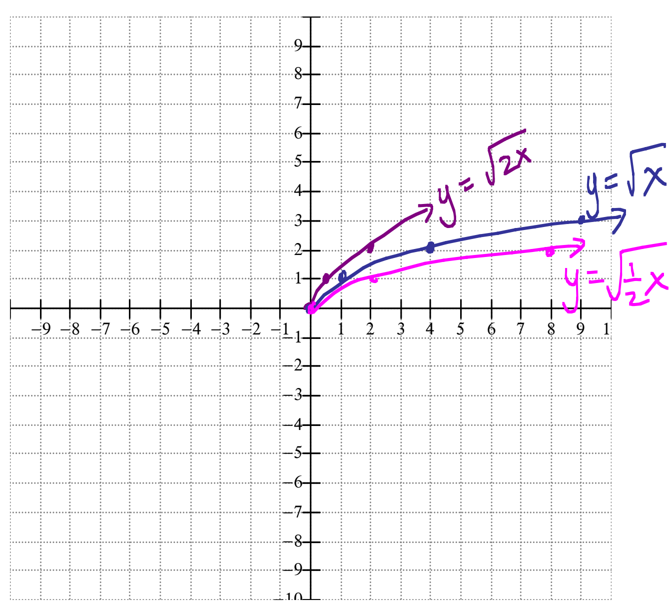
$y = \sqrt{2(2)}$
$= \sqrt{4}$
$= 2$

$\boxed{y = \sqrt{\frac{1}{2}x}}$

x	y
0	0
2	1
8	2

$y = \sqrt{\frac{1}{2}(2)}$
$y = \sqrt{1}$
$= 1$

$y = \sqrt{\frac{1}{2}(8)}$
$= \sqrt{4}$
$= 2$



In general, for any function y =f(x), the graph of y = f(**b**x), where **b** is any real number results in a <u>horizontal stretch</u>. Mapping Notation: $(x, y) \to (\frac{x}{b}, y)$

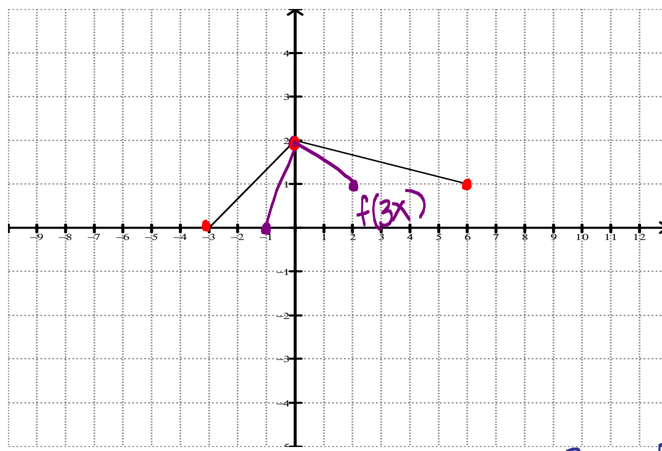
Ex. #5: Given the graph of y = f(x),

- Transform the graph of f(x) to sketch the graph of g(x) = f(3x)
- Describe the transformation
- State any invariant points: point that does not change
- State the domain and range of the new function

Horizontal stretch by a factor of $\frac{1}{3}$

2	~~6~~	1
0	~~0~~	2
-1	~~-3~~	0

divide x-values by 3
or
multiply by $\frac{1}{3}$



(0,2) Invariant point

$\{x: -1 \le x \le 2 \quad x \in \mathbb{R}\}$ $[-1, 2]$

$\{y: 0 \le y \le 2 \quad y \in \mathbb{R}\}$

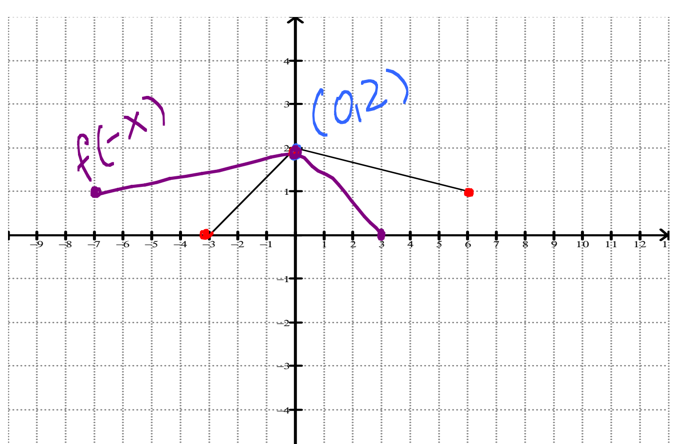
Ex. #6: Given the graph of y = f(x),

- Transform the graph of f(x) to sketch the graph of g(x) = f(-x)
- Describe the transformation
- State any invariant points
- State the domain and range of the new function

f(-1x)

-6	~~6~~	1
0	~~0~~	2
3	~~-3~~	0

divide x-values by -1



Reflection over y-axis
(0,2) invariant point

$\{x: -6 \le x \le 3 \quad x \in \mathbb{R}\}$

$\{y: 0 \le y \le 2 \quad y \in \mathbb{R}\}$

Ex. #7: Given f(x) = (x – 2)(x + 3) use transformations to determine the zeroes of each of the following functions.

(a) y = f(2x) divide x's by 2

$(2,0) \to (1,0)$

$(-3,0) \to (-\frac{3}{2}, 0)$

(b) y = f(-x) Reflection over y-axis
divide x's by -1

$(2,0) \to (-2,0)$

$(-3,0) \to (3,0)$

(c) y = 3f(x) Multiply y's by 3

$(2,0) \to (2,0)$ No change

$(-3,0) \to (-3,0)$

(d) y = -f(x) Reflection over x-axis

$(2,0) \to (2,0)$

$(-3,0) \to (-3,0)$

zeroes of f(x)

$0 = (x-2)(x+3)$

$0 = x - 2 \qquad 0 = x + 3$

$x = 2 \qquad x = -3$

$(2,0)\ (-3,0)$