

1.3 Combining Transformations

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Pre-Calculus 12

1.3 Combining Transformations

General Transformation Equation

$$y = af(b(x-h)) + k \quad \text{or} \quad y - k = af(b(x-h))$$

- a vertical stretch factor of $|a|$, $a < 0$ reflection over x-axis
- b horizontal stretch factor of $|\frac{1}{b}|$, $b < 0$ reflection over y-axis
- h horizontal translation
- k vertical translation

To simplify the procedure of combining transformations we perform transformations in the following order:

- stretches
- reflections
- translations

Ex. #1: Describe the transformations using an appropriate order to graph the new function from the original $y = f(x)$. Then, give the mapping for each.

(a) $y = -f(2(x+3)) - 1$

reflection over x-axis
 horizontal stretch factor $\frac{1}{2}$
 horizontal translation left 3
 vertical translation down 1

$$(x, y) \rightarrow \left(\frac{x}{2} - 3, -y - 1\right)$$

(b) $y - 3 = 2f(-3x - 6)$

$y = 2f(-3(x+2)) + 3$
 vertical stretch factor of 2
 Reflection over y-axis
 Horizontal stretch factor $\frac{1}{3}$
 Horizontal translation left 2
 vertical translation up 3

$$(x, y) \rightarrow \left(\frac{x}{-3} - 2, y + 3\right)$$

Note: Functions of the form $f(bx+h)$ must be in Factored form.

$$y = f(10x - 20)$$

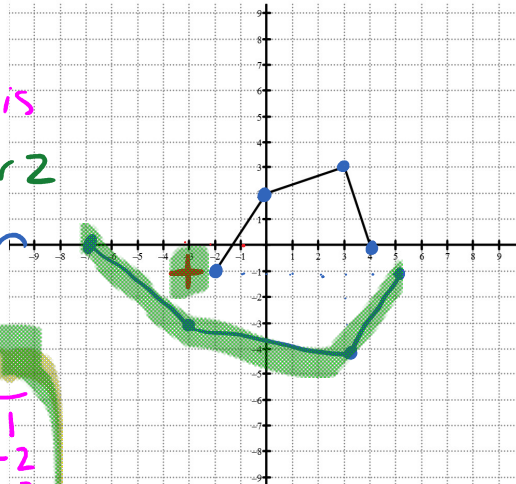
$$y = f(10(x - 2))$$

Ex. #2: Given the graph of $y = f(x)$, describe the transformations in an appropriate order needed to sketch the new graph. Sketch the transformed graph.

$$y = -f\left(\frac{1}{2}(x+3)\right) - 1$$

- reflection over x-axis
- Horizontal stretch factor 2
- Horizontal translation left 3
- Vertical translation down 1

Fake Axis
 $\leftarrow 3$
 $\downarrow 1$



old	
-2	-1
3	2
0	3
4	0

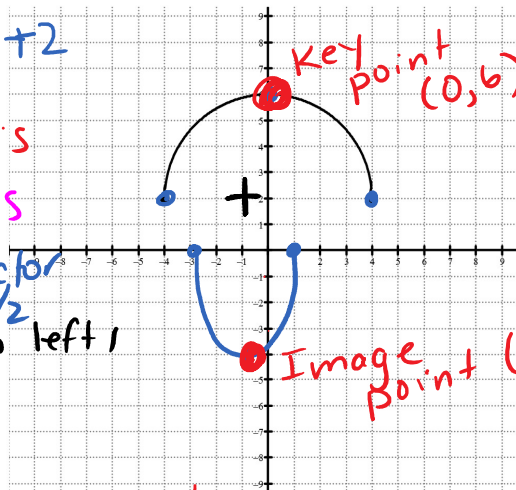
$a = -1$ $b = \frac{1}{2}$
 Mult y's by (-1) Divide x's by $\frac{1}{2}$

-4	1
0	-2
6	-3
0	0

Ex. #3: : Given the graph of $y = f(x)$, describe the transformations in an appropriate order needed to sketch the new graph. Sketch the transformed graph.

$$y - 2 = -f(-2(x+1)) + 2$$

- Reflection over x-axis
- Reflection over y-axis
- Horizontal stretch factor of $\frac{1}{2}$
- Horizontal translation left 1
- Vertical translation up 2



old	
-4	2
0	6
4	2

$a = -1$
 Mult y's by (-1)

$b = -2$
 divide x's by (-2)

2	-2
0	-6
-2	-2

Ex. #4: The ^{old} key point (4, -6) is on the graph of $y = f(x)$. What is the image point under each transformation of the graph of $f(x)$?

(a) $y - 6 = 2f(2x - 4)$

$y = 2f(2(x-2)) + 6$

$4 \mid -6$

$a=2$
Mult y's
by 2

$b=2$
divide x's
by 2

$2 \mid -12$

→ 2
Add 2
to x's

↑ 6
Add 6
to y's

$4 \mid -6$

(b) $y = -\frac{2}{3}f(x+1) - 3$

$4 \mid -6$

$a = -\frac{2}{3}$
Mult y's
by $(-\frac{2}{3})$

$-6 \cdot (-\frac{2}{3}) = \frac{12}{3} = 4$

$4 \mid 4$

← 1
Subtract 1
from x's
↓ 3
subtract 3
from y's

$3 \mid 1$

Ex. #5: The graph of $y = g(x)$ represents a transformation of the graph $y = f(x)$. Determine an equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

reflection over y-axis
 $b < 0$

$a > 0$ (No reflection
over x-axis)

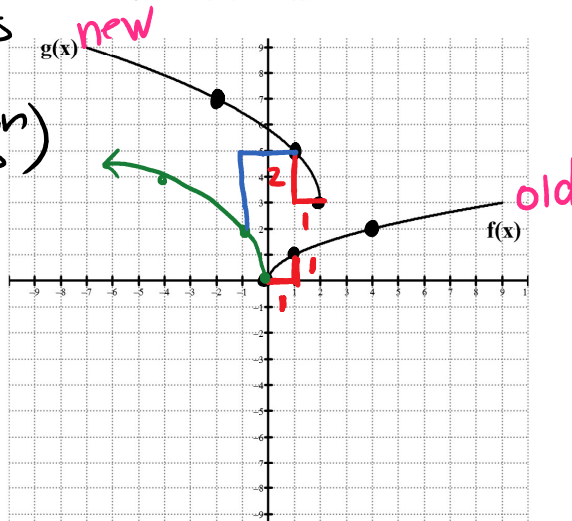
No Horizontal
vertical stretch factor
of 2

$a = 2$ $b = -1$

Key points

0	0
1	1
4	2

0	0
-1	2
-4	4



Translate up 3 right 2
 $y = 2f(-1(x-2)) + 3$