1.4 Inverses: Part I

The inverse of a relation can be found by interchanging the x and y coordinates of the original function.

$$(x, y) \rightarrow (y, x)$$

For every (x, y) of a relation, there is an ordered pair (y, x) on the inverse of that relation.

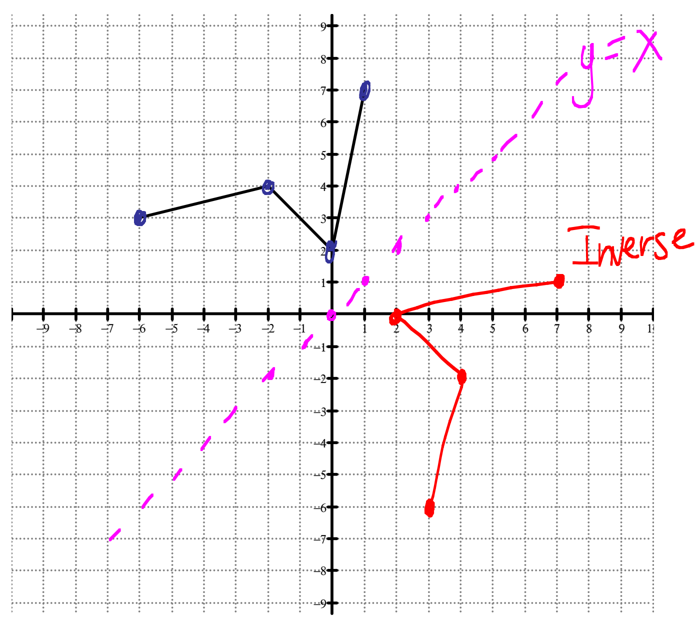
Ex. #2: Given the graph of the relation below sketch the graph of its inverse.

Original

-6	3
-2	4
0	2
1	7

Inverse

3	-6
4	-2
2	0
7	1



Ex. #3: On the above graph sketch the line y = x.
What do you notice about the graphs with respect to the line y = x?

They are reflections of each other over y=x

Ex. #4: Is the graph of the original relation a function? How do you know?

Yes. Original relation is a function
One y-value for each x-value

Is the graph of the inverse a function? How could you tell without graphing the inverse whether it would be a function?

No, Not a function.
Some x-values that have 2 different y-values.

The graph of a relation and its inverse are __Reflections__ of each other in the line y = x.

Horizontal Line Test:

- A test used to determine whether the graph of an inverse relation will be a __function__.
- If a horizontal line intersects a graph in __more than one place__ then the $\boxed{\text{inverse of the relation is not a function.}}$

The inverse of a function y = f(x) may be written in the form __$x = f(y)$__.

When the inverse of a function is itself a function then we use the notation

if the inverse is a function
the the inverse can be written as $f^{-1}(x)$

<u>Ex. #5</u>: Consider the function y = f(x) sketched below:

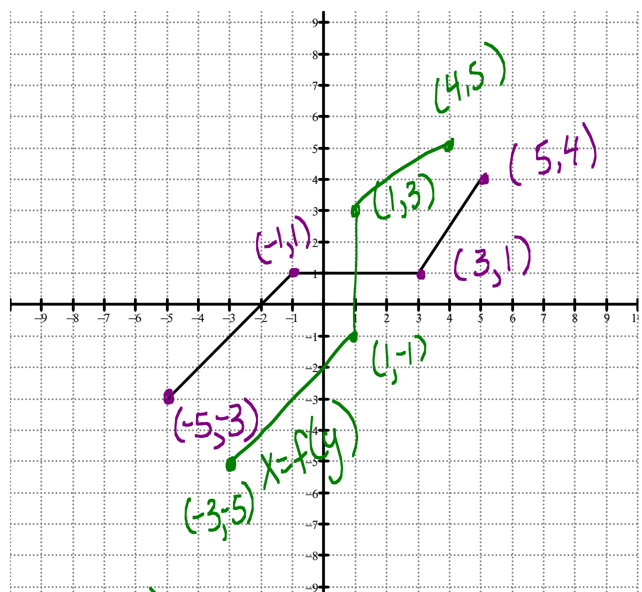
(a) Without graphing will the inverse graph be a function?

Horizontal line test

Not a function

(b) Sketch the graph of x = f(y).

(c) State the domain and range for both the original and the inverse.



$y = f(x)$

$\{x : -5 \le x \le 5 \quad x \in \mathbb{R}\}$

$\{y : -3 \le y \le 4 \quad y \in \mathbb{R}\}$

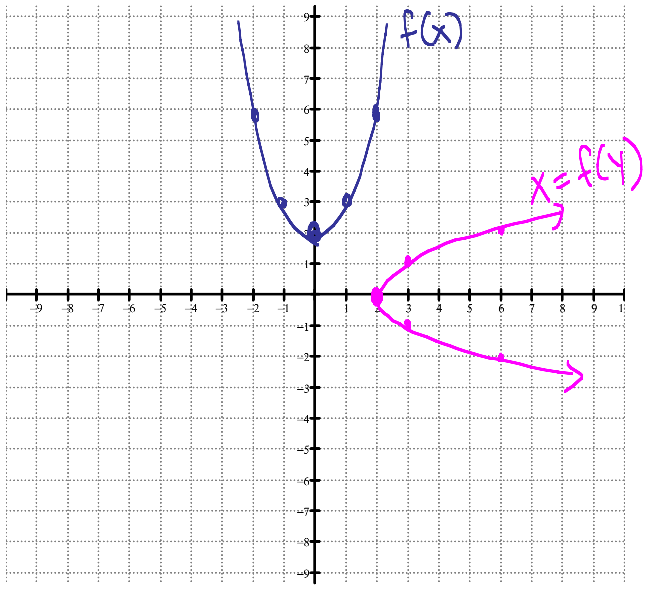
$x = f(y)$

$\{x : -3 \le x \le 4 \quad x \in \mathbb{R}\}$

$\{y : -5 \le y \le 5 \quad y \in \mathbb{R}\}$

Ex. #6: Consider the function $\boxed{f(x) = x^2 + 2.}$

(a) Graph the function f(x). Is the inverse of f(x) a function?



The inverse of f(x) is not a function. f(x) fails the Horizontal line test.

(b) Graph the inverse of f(x)

(c) State the domain and range of f(x) and its inverse.

f(x) $\{x : x \in \mathbb{R}\}$

$\{y : y \geq 2\ y \in \mathbb{R}\}$

x=f(y) $\{x : x \geq 2\ x \in \mathbb{R}\}$

$\{y : y \in \mathbb{R}\}$

(d) Restrict the domain of f(x) so that its inverse will be a function.

$x \geq 0$ We only get the right side of parabola.

(e) Sketch f(x) with its restricted domain and its inverse.

$f(x) = x^2 + 2 \quad x \geq 0$