Pg 519 10.1

#3. $y' - 8x = 0$

$\frac{dy}{dx} = 8x$

$1\,dy = 8x\,dx$

$\int 1\,dy = \int 8x\,dx$

$y = \frac{8x^2}{2} + C$

$y = 4x^2 + C$

4. $y\,y' + 4x = 0$

$y\frac{dy}{dx} = -4x$

$y\,dy = -4x\,dx$

$\int y\,dy = \int -4x\,dx$

$\frac{1}{2}y^2 = -\frac{4}{2}x^2 + C$

$y^2 = -4x^2 + C$

$y = \pm\sqrt{-4x^2 + C}$

$y = \pm\sqrt{C - 4x^2}$

5. $y' + 4xy = 0$

$\frac{dy}{dx} = -4xy$

$\frac{1}{y}dy = -4x\,dx$

$\int \frac{1}{y}dy = \int -4x\,dx$

$\ln|y| = -\frac{4}{2}x^2 + C$

$\ln|y| = -2x^2 + C$

$e^{-2x^2+C} = y$

$e^{-2x^2} \cdot e^C = y$

e^C is a constant

$Ce^{-2x^2} = y$

13. $y' + 4xy^2 = 0$

$\frac{dy}{dx} = -4xy^2$

$\frac{1}{y^2}\,dy = -4x\,dx$

$\int y^{-2}\,dy = \int -4x\,dx$

$\frac{y^{-1}}{-1} = -\frac{4}{2}x^2 + C$

$-\frac{1}{y} = -2x^2 + C$

$\frac{1}{y} = 2x^2 + C$

$y = \frac{1}{2x^2 + C}$

15. $\frac{dy}{dt} - 20t^4e^{-y} = 0$

$\frac{dy}{dt} = 20t^4e^{-y}$

$e^y\,dy = 20t^4\,dt$

$\int e^y\,dy = \int 20t^4\,dt$

$e^y = \frac{20}{5}t^5 + C$

$e^y = 4t^5 + C$

$\ln e^y = \ln|4t^5 + C|$

$y = \ln|4t^5 + C|$

19. $\sqrt{1-x^2}\,y' = xy$

$\sqrt{1-x^2}\,\frac{dy}{dx} = xy$

$\frac{1}{y}\,dy = \frac{x}{\sqrt{1-x^2}}\,dx$

$\int \frac{1}{y}\,dy = \int \frac{x}{\sqrt{1-x^2}}\,dx$

$u = 1 - x^2$

$du = -2x\,dx$

$\frac{du}{-2} = dx$

$\ln|y| = \int \frac{1}{\sqrt{u}} \cdot \frac{du}{-2}$

$\ln|y| = -\frac{1}{2}\left(\frac{2}{1}\right)u^{1/2} + C$

$\ln|y| = -\sqrt{1-x^2} + C$

$e^{-\sqrt{1-x^2}+C} = y$

$e^{-\sqrt{1-x^2}} \cdot e^C = y$

$y = Ce^{-\sqrt{1-x^2}}$

21. $y y' = x$

$$y \frac{dy}{dx} = x$$

$$y\,dy = x\,dx$$

$$\int y\,dy = \int x\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$$

$$y^2 = x^2 + C$$

$$y = \pm\sqrt{x^2 + C}$$

23. $\frac{dx}{dt} = (t+1)(x^2+1)$

$$\frac{1}{x^2+1}\,dx = (t+1)\,dt$$

$$\int \frac{1}{x^2+1}\,dx = \int (t+1)\,dt$$

$$\arctan x = \frac{1}{2}t^2 + t + C$$

$$x = \tan\left(\frac{1}{2}t^2 + t + C\right)$$

29. $y' + 2y = 0 \quad y(\ln 5) = 3$

$$\frac{dy}{dx} = -2y$$

$$\frac{1}{y}\,dy = -2\,dx$$

$$\int \frac{1}{y}\,dy = \int -2\,dx$$

$$\ln|y| = -2x + C$$

$$e^{-2x+C} = y$$

$$\boxed{Ce^{-2x} = y}$$

$$Ce^{-2\ln 5} = 3$$

$$C = \frac{3}{(e^{\ln 5})^{-2}}$$

$$C = 3(e^{\ln 5})^2$$

$$C = 3(5)^2$$

$$C = 75$$

$$y = 75e^{-2x}$$

31. $y\,y' = x e^{-y^2}$ $\quad y(0) = -2$

$y \frac{dy}{dx} = x e^{-y^2}$

$\frac{y\,dy}{e^{-y^2}} = x\,dx$

$\int y e^{y^2} dy = \int x dx$

$u = y^2$

$du = 2y\,dy$

$\frac{du}{2} = y\,dy$

$\int e^u \cdot \frac{du}{2} = \int x dx$

$\frac{1}{2} e^u = \frac{1}{2} x^2 + C$

$e^{y^2} = x^2 + C$

$\ln e^{y^2} = \ln|x^2 + e^4|$

$y^2 = \ln|x^2 + e^4|$

$y = \pm\sqrt{\ln|x^2 + e^4|}$

$y(0) = -2$

$e^{(-2)^2} = 0^2 + C$

$e^4 = C$

Since $y(0) = -2$

↑ negative value only

Use $-\sqrt{\ }$

$y = -\sqrt{\ln(x^2 + e^4)}$

33. $y' = (x-1)(y-2)$ $\quad y(2) = 4$

$$\frac{1}{y-2}\frac{dy}{dx} = (x-1)$$

$$\int \frac{1}{y-2}\,dy = \int (x-1)\,dx$$

$$\ln|y-2| = \frac{1}{2}x^2 - x + C$$

$$e^{\frac{1}{2}x^2 - x + C} = y - 2$$

$$e^C e^{\frac{1}{2}x^2 - x} + 2 = y$$

$$C e^{\frac{1}{2}x^2 - x} + 2 = y$$

$$C e^{\frac{1}{2}(2)^2 - 2} + 2 = 4$$

$$C e^0 = 2$$

$$C = 2$$

$$y = 2e^{\frac{1}{2}x^2 - x} + 2$$

37. $\frac{dy}{dt} = ye^{-t}$ $\quad y(0) = 1$

$\frac{1}{y}\,dy = e^{-t}\,dt$

$\int \frac{1}{y}\,dy = \int e^{-t}\,dt$

$\ln|y| = -e^{-t} + c$

$e^{-e^{-t}+c} = y$

$e^{c} \cdot e^{-e^{-t}} = y$

$C \cdot e^{-e^{-t}} = y$ $\quad y(0) = 1$

$C \cdot e^{-e^{0}} = 1$

$C \cdot e^{-1} = 1$

$C = \frac{1}{e^{-1}}$

$C = e$

$y = e \cdot e^{-e^{-t}}$

$y = e^{1} \cdot e^{-e^{-t}}$

$y = e^{1-e^{-t}}$