

# 10.1 Sums and Differences of Functions

Wednesday, January 5, 2022 10:39 AM

## 10.1 Sums and Differences of Functions

### Review

1. Given  $f(x) = x^2 - 7x + 10$ .

a)  $f(3) = (3)^2 - 7(3) + 10$   
 $f(3) = 9 - 21 + 10$   
 $f(3) = -2$

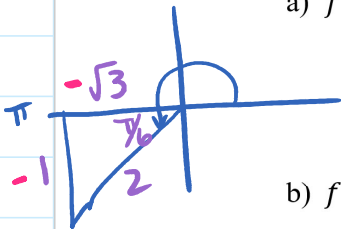
b)  $f(-2) = (-2)^2 - 7(-2) + 10$   
 $f(-2) = 4 + 14 + 10$   
 $f(-2) = 28$

c)  $f(2m) = (2m)^2 - 7(2m) + 10$   
 $f(2m) = 4m^2 - 14m + 10$

d)  $f(-x) = (-x)^2 - 7(-x) + 10$   
 $f(-x) = x^2 + 7x + 10$

2. Given  $f(x) = \sin x$ , graph  $f(x)$  and find the following

a)  $f\left(\frac{7\pi}{6}\right) = \sin \frac{7\pi}{6} = -\frac{1}{2}$   
 $\text{ref } \angle = \frac{7\pi}{6} - \frac{6\pi}{6} = \frac{\pi}{6}$



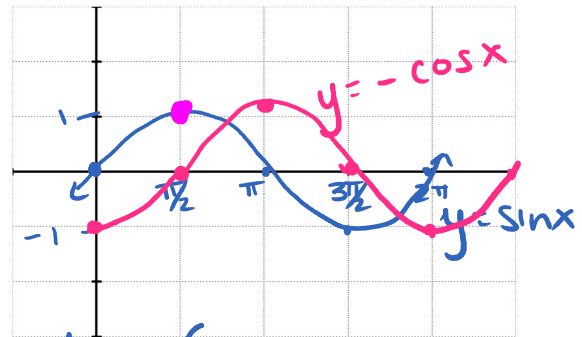
b)  $f\left(x - \frac{\pi}{2}\right) = \sin\left(x - \frac{\pi}{2}\right)$

$f\left(x - \frac{\pi}{2}\right) = \sin x \cos \frac{\pi}{2} - \cos x \sin \frac{\pi}{2}$

$f\left(x - \frac{\pi}{2}\right) = \sin x (0) - \cos x (1)$   
 $= -\cos x$

c) How does  $f\left(x - \frac{\pi}{2}\right)$  compare to  $f(x)$

$\sin\left(x - \frac{\pi}{2}\right)$



$\sin x$   
 Horizontal translation right  $\frac{\pi}{2}$   
 Phase shift

d) Graph  $f\left(x - \frac{\pi}{2}\right)$

New functions can be formed by adding or subtracting functions.

Sums of Functions

$$h(x) = f(x) + g(x)$$

$$h(x) = (f+g)(x)$$

Differences of Functions

$$h(x) = f(x) - g(x)$$

$$h(x) = (f-g)(x)$$

Ex. #1 Use  $f(x) = 5x - 1$  and  $g(x) = 2x$  to find the following:

a)  $(f + g)(2)$   
 $= f(2) + g(2)$   
 $= 5(2) - 1 + 2(2)$   
 $= 10 - 1 + 4$   
 $= 13$

b)  $(f - g)(-1)$   
 $= f(-1) - g(-1)$   
 $= 5(-1) - 1 - 2(-1)$   
 $= -5 - 1 + 2$   
 $= -4$

c)  $h(x) = (f + g)(x)$   
 $h(x) = f(x) + g(x)$   
 $h(x) = 5x - 1 + 2x$   
 $h(x) = 7x - 1$

d) Use  $h(x)$  to find  $(f + g)(10)$   
 $(f + g)(10) = h(10)$   
 $h(10) = 7(10) - 1$   
 $h(10) = 70 - 1$   
 $h(10) = 69$

Ex. #2: Given  $f(x) = \frac{2}{x}$  and  $g(x) = \frac{3}{x+2}$  find the following functions.

a)  $(f + g)(x)$

b)  $(f - g)(x)$

$(f + g)(x) = f(x) + g(x)$   
 $= \frac{2}{x} + \frac{3}{x+2}$   
 $\rightarrow$   
 $x \neq 0$   
 $x \neq -2$   
 $= \frac{2(x+2)}{x(x+2)} + \frac{3x}{(x+2)x}$   
 $= \frac{2x+4+3x}{x(x+2)}$   
 $= \frac{5x+4}{x(x+2)}$

$(f - g)(x) = f(x) - g(x)$   
 $= \frac{2}{x} - \frac{3}{x+2}$   
 $x \neq 0$   
 $x \neq -2$   
 $= \frac{2(x+2)}{x(x+2)} - \frac{3x}{(x+2)x}$   
 $= \frac{2x+4-3x}{x(x+2)}$   
 $= \frac{-x+4}{x(x+2)}$

Ex. #3: Given  $f(x) = 3\sqrt{x+2}$  and  $g(x) = \sqrt{x+2} - 5$  find the following:

**Ex. #3:** Given  $f(x) = 3\sqrt{x+2}$  and  $g(x) = \sqrt{x+2} - 5$  find the following:

a)  $(f+g)(x)$   
 $= f(x) + g(x)$   
 $= 3\sqrt{x+2} + \sqrt{x+2} - 5$   
 $= 4\sqrt{x+2} - 5$

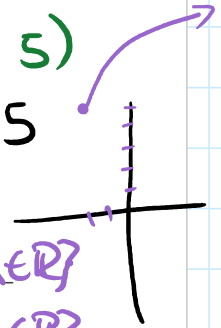
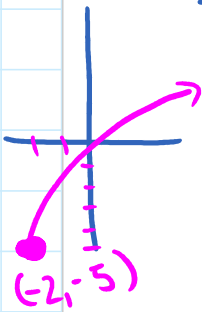
Domain:  $\{x \mid x \geq -2, x \in \mathbb{R}\}$

Range:  $\{y \mid y \geq -5, y \in \mathbb{R}\}$

b)  $(f-g)(x)$   
 $= f(x) - g(x)$   
 $= 3\sqrt{x+2} - (\sqrt{x+2} - 5)$   
 $= 3\sqrt{x+2} - \sqrt{x+2} + 5$   
 $= 2\sqrt{x+2} + 5$

Domain:  $\{x \mid x \geq -2, x \in \mathbb{R}\}$

Range:  $\{y \mid y \geq 5, y \in \mathbb{R}\}$



**Ex. #4:** Find the equation of  $h(x) = f(x) + g(x)$  then find the domain and range of  $h(x)$ .  $f(x) = 2x^2$  and  $g(x) = 4x - 1$ . Sketch  $h(x)$ .

$h(x) = f(x) + g(x)$

$h(x) = 2x^2 + 4x - 1$

Complete the square

$h(x) = 2(x^2 + 2x) - 1$

$h(x) = 2(x^2 + 2x + \square) - 1 - 2\square$

$\left(\frac{2}{2}\right)^2 = 1^2 = 1$

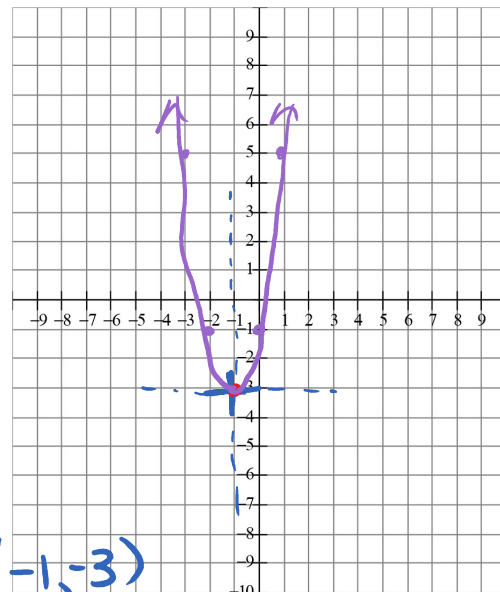
$h(x) = 2(x+1)^2 - 1 - 2$

$h(x) = 2(x+1)^2 - 3$

vertex  $(-1, -3)$

Domain:  $\{x \mid x \in \mathbb{R}\}$

Range:  $\{y \mid y \geq -3, y \in \mathbb{R}\}$



$y = x^2$

0	0
1	1
2	4

$a = 2$

$y = 2x^2$

0	0
1	2
2	8