10.2 Slope Fields

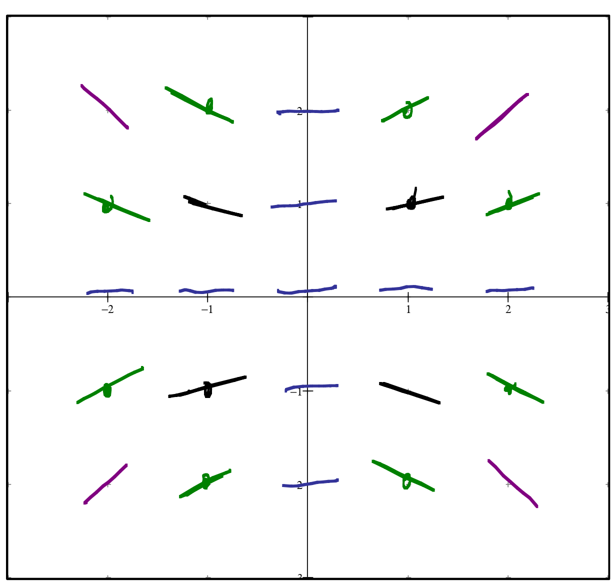
<u>Slope field:</u> A graphical representation of the flow of tangent lines to the family of solutions for a differential equation.

<u>Constructing a slope field:</u>
Substitute points (x, y) into the differential equation to find slopes of tangent lines

1. Construct a slope field for $\frac{dy}{dx} = \frac{xy}{4}$ for $x \in [-2,2]$ and $y \in [-2,2]$

y-value

x-value \ $\frac{dy}{dx}$	-2	-1	0	1	2
-2	$\frac{(-2)(-2)}{4} = 1$	$\frac{(-2)(-1)}{4} = \frac{1}{2}$	0	$\frac{(-2)(1)}{4} = -\frac{1}{2}$	$\frac{(-2)(2)}{4} = -1$
-1	$\frac{(-1)(-2)}{4} = \frac{1}{2}$	$\frac{(-1)(-1)}{4} = \frac{1}{4}$	0	$\frac{(-1)(1)}{4} = -\frac{1}{4}$	$\frac{(-1)(2)}{4} = -\frac{1}{2}$
0	0	0	0	0	0
1	$\frac{(1)(-2)}{4} = -\frac{1}{2}$	$\frac{(1)(-1)}{4} = -\frac{1}{4}$	0	$\frac{(1)(1)}{4} = \frac{1}{4}$	$\frac{(1)(2)}{4} = \frac{1}{2}$
2	$\frac{2(-2)}{4} = -1$	$\frac{(2)(-1)}{4} = -\frac{1}{2}$	0	$\frac{(2)(1)}{4} = \frac{1}{2}$	$\frac{2(2)}{4} = 1$



AP Calculus

<u>Reading A Slope Field:</u>

① Examine along vertical lines: if slopes are the same then $\frac{dy}{dx}$ does not depend on y.

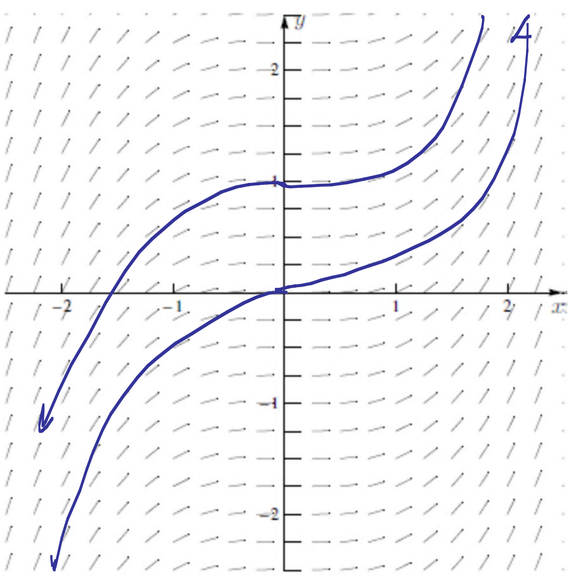
② Along Horizontal lines if slopes are the same then $\frac{dy}{dx}$ does not depend on x.

③ Quad ① if slope segments are all ⊕ $\frac{dy}{dx}$ is not negative

④ When x get bigger if dy/dx gets bigger dy/dx relates directly to x

2. Consider the slope field in the window $x \in [-2.5, 2.5]$ and $y \in [-2.5, 2.5]$

a) What can you interpret from the slope segments.



- On vertical lines $\frac{dy}{dx}$ does not change. No y in $\frac{dy}{dx}$
- $\frac{dy}{dx}$ is always ⊕
- as x gets bigger $\frac{dy}{dx}$ gets bigger) (in Quad ①)

b) Which of the following is most likely the differential equation

A) $\frac{dy}{dx} = 0.5xy$ B) $\frac{dy}{dx} = \frac{x^2}{y}$ C) $\frac{dy}{dx} = 0.5x^2$

c) Solve the differential equation

$$\frac{dy}{dx} = 0.5x^2$$

$$dy = \frac{1}{2}x^2\,dx$$

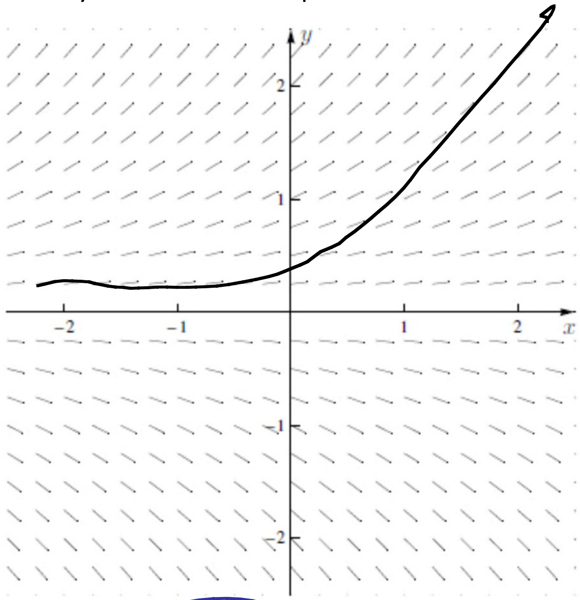
$$\int dy = \int \frac{1}{2}x^2\,dx$$

$$y = \frac{1}{2}\cdot\frac{1}{3}x^3 + C$$

$$y = \frac{x^3}{6} + C$$

AP Calculus

3. Determine which of the following differential equations is the solution to the slope field. Then solve your differential equation.



Horizontal segments have same slope

No x's for $\frac{dy}{dx}$

Need y's and no negative sign for $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{2}y$

$\frac{1}{y}dy = \frac{1}{2}dx$

$\int \frac{1}{y}dy = \int \frac{1}{2}dx$

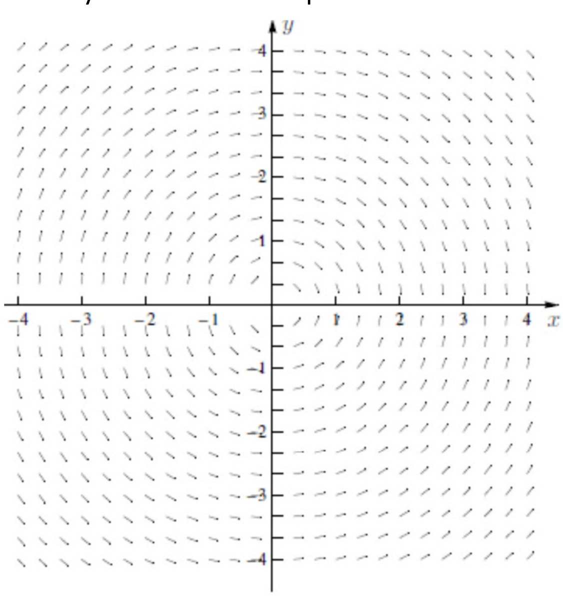
$\ln|y| = \frac{1}{2}x + c$

$e^{\frac{1}{2}x + c} = y$

$y = Ce^{\frac{1}{2}x}$

A) $\frac{dy}{dx} = 0.5y$ B) $\frac{dy}{dx} = \frac{0.2x}{y}$ C) $\frac{dy}{dx} = xy$ D) $\frac{dy}{dx} = x + y$ E) $\frac{dy}{dx} = \frac{1}{x}$

4. Determine which of the following differential equations is the solution to the slope field. Then solve your differential equation.



Need x and y as slopes change along horizontal and vertical lines

Quad ① $\frac{dy}{dx}$ neg } $\frac{dy}{dx}$ needs a ⊖

$\frac{dy}{dx} = -\frac{x}{y}$

$y\,dy = -x\,dx$

$\int y\,dy = \int -x\,dx$

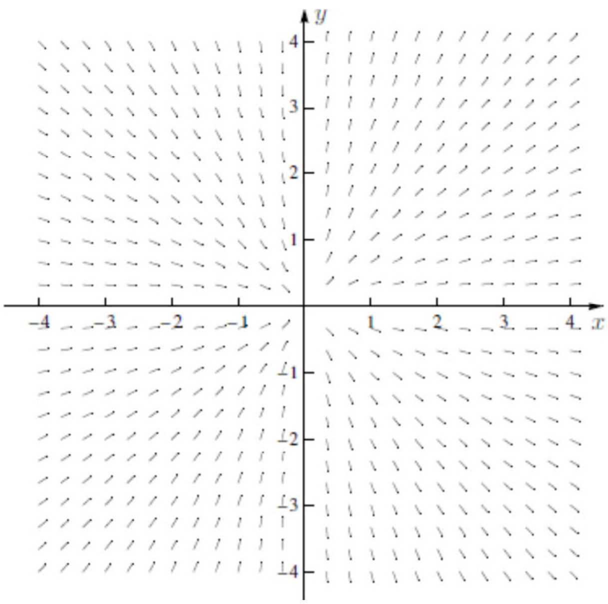
$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + c$

$y^2 = -x^2 + c$

$x^2 + y^2 = C$

A) $\frac{dy}{dx} = x^2$ B) $\frac{dy}{dx} = \frac{y}{x}$ C) $\frac{dy}{dx} = -y$ D) $\frac{dy}{dx} = -\frac{x}{y}$ E) $\frac{dy}{dx} = x^2 + y^2$

5. Determine which of the following differential equations is the solution to the slope field. Then solve your differential equation.



Need x and y as slopes change along vertical and horizontal lines.

Quad ① slopes ⊕ $\frac{dy}{dx}$ is ⊕

A) $\frac{dy}{dx} = x + y$ B) $\frac{dy}{dx} = x - y$ C) $\frac{dy}{dx} = x^2$ D) $\frac{dy}{dx} = 2y$ E) $\frac{dy}{dx} = \frac{y}{x}$

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\frac{1}{y}\,dy = \frac{1}{x}\,dx$$

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\ln|y| = \underbrace{\ln|x| + C}$$

$$e^{\ln|x| + C} = y$$

$$e^{\ln|x|} \circ e^{C} = y$$

$$y = Cx$$