10.2 Products and Quotients of Functions

Wednesday, January 5, 2022 10:40 AM

10.2 Products, Quotients and Combinations of Functions

Review

1. Given $f(x) = 2x^2 + 3x$ and $g(x) = 5 - x$ find the following.

a) $4f(x)$

$= 4(2x^2+3x)$

$= 8x^2 + 12x$

b) $2g(x) - f(x)$

$= 2(5-x) - (2x^2+3x)$

$= 10 - 2x - 2x^2 - 3x$

$= -2x^2 - 5x + 10$

c) $f(2) + 4g(2)$

$= 2(2)^2 + 3(2) + 4[5-2]$

$= 8 + 6 + 4(3)$

$= 8 + 6 + 12$

$= 26$

d) $2(f(x) + g(x))$

$= 2(2x^2 + 3x + 5 - x)$

$= 2(2x^2 + 2x + 5)$

$= 4x^2 + 4x + 10$

Products of Functions

$h(x) = f(x)g(x)$

$h(x) = (fg)(x)$

Quotients of Functions

$h(x) = \frac{f(x)}{g(x)}$

$h(x) = \left(\frac{f}{g}\right)(x)$

Ex. #1 Given $f(x) = x^2 - 4$ and $g(x) = x - 1$ find the following:

b) $(fg)(-1)$

$= f(-1) \cdot g(-1)$

$= [(-1)^2 - 4] \cdot [-1-1]$

$= (-3) \cdot (-2) = 6$

c) $\left(\frac{f}{g}\right)(0)$

$= \frac{f(0)}{g(0)}$

$= \frac{0^2 - 4}{0 - 1} = \frac{-4}{-1} = 4$

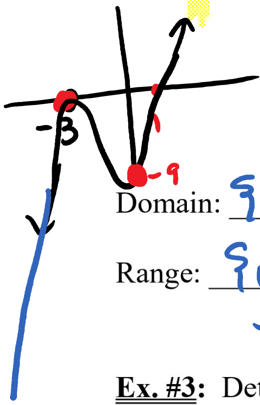
Ex. #2: $f(x) = x + 3$ and $g(x) = x^2 + 2x - 3$ Determine the equations for $h(x)$ and $k(x)$ then state the domain and range of the new functions.

a) $h(x) = f(x)g(x)$

$h(x) = (x+3)(x^2+2x-3)$

$h(x) = x^3 + 2x^2 - 3x + 3x^2 + 6x - 9$

$h(x) = 1x^3 + 5x^2 + 3x - 9$



x^2+2x-3

$(x+3)(x-1)$

Domain: $\{x \mid x \in \mathbb{R}\}$

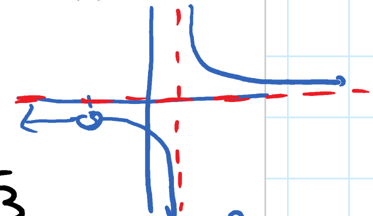
Range: $\{y \mid y \in \mathbb{R}\}$

b) $k(x) = \frac{f(x)}{g(x)}$

$K(x) = \dfrac{x+3}{x^2+2x-3}$

$K(x) = \dfrac{\cancel{x+3}}{\cancel{(x+3)}(x-1)}$

$K(x) = \dfrac{1}{x-1} + 0$



point of discontinuity at $x=-3$

vertical asymptote $x=1$

$K(x) = \dfrac{1}{x-1}$

$K(-3) = \dfrac{1}{-3-1}$

$K(-3) = \dfrac{1}{-4}$

Domain: $\{x \mid x \neq -3, x \neq 1 \ x \in \mathbb{R}\}$

Range: $\{y \mid y \neq 0, y \neq -\frac{1}{4} \ y \in \mathbb{R}\}$

Ex. #3: Determine the equation of $h(x) = \frac{f(x)}{g(x)}$ and graph $h(x)$ if $f(x) = 2x - 8$ and $g(x) = x^2 - 3x - 4$.

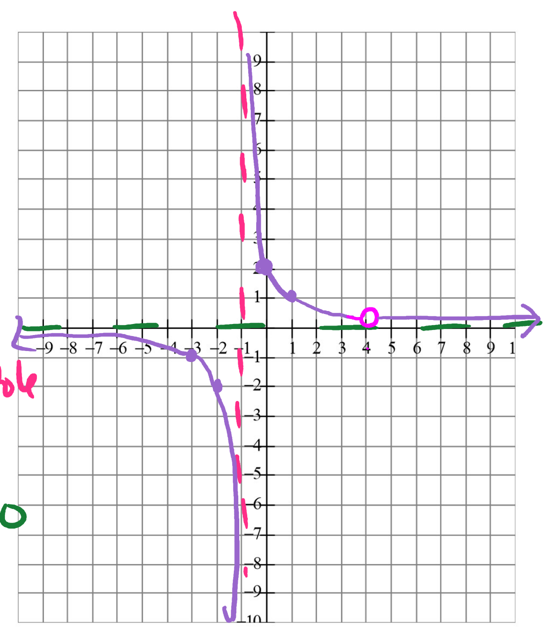
$h(x) = \dfrac{2x-8}{x^2-3x-4}$

$h(x) = \dfrac{2\cancel{(x-4)}}{\cancel{(x-4)}(x+1)}$

point of discontinuity at $x=4$

$h(x) = \dfrac{2}{x+1} + 0$ vertical asymptote $x=-1$

Horizontal asymptote $y=0$



$y = \frac{1}{x}$

x	y
1	1

$y = \frac{2}{x}$

x	y
1	2
2	1
-1	-2
-2	-1

Ex. #5: Given that $f(x) = \frac{1}{x+3}$ and $g(x) = \frac{3}{x}$. Determine the equations for $h(x)$ and $k(x)$, state any restrictions.

a) $h(x) = \frac{f(x)}{g(x)}$

$$h(x) = \frac{\frac{1}{x+3}}{\frac{3}{x}}$$

$x \neq -3$

$x \neq 0$

$$h(x) = \frac{1}{x+3} \cdot \frac{x}{3}$$

$$h(x) = \frac{x}{3(x+3)}$$

b) $k(x) = f(x)g(x)$

$$k(x) = \frac{1}{x+3} \cdot \frac{3}{x}$$

$x \neq 0$

$x \neq -3$

$$k(x) = \frac{3}{(x+3)x}$$

$$k(x) = \frac{3}{x(x+3)}$$

Ex. #6: Given that $f(x) = 2\sqrt{x} - 3$ and $g(x) = \sqrt{x} + 1$. Determine the equations for $h(x)$ and $k(x)$, state any restrictions.

a) $h(x) = \frac{f(x)}{g(x)}$

$$h(x) = \frac{2\sqrt{x} - 3}{\sqrt{x} + 1} \cdot \frac{(\sqrt{x} - 1)}{(\sqrt{x} - 1)}$$

$$h(x) = \frac{2x - 2\sqrt{x} - 3\sqrt{x} + 3}{x - \sqrt{x} + \sqrt{x} - 1}$$

$$h(x) = \frac{2x - 5\sqrt{x} + 3}{x - 1}$$

$x \neq 1$ denominator

$\sqrt{x}$

$x \geq 0$ radical

b) $k(x) = f(x)g(x)$

$$k(x) = (2\sqrt{x} - 3)(\sqrt{x} + 1)$$

$$k(x) = 2x + 2\sqrt{x} - 3\sqrt{x} - 3$$

$$k(x) = 2x - \sqrt{x} - 3$$

$\sqrt{x}$ $x \geq 0$