

10.3 Function Composition Part 1

Thursday, December 13, 2018 8:56 AM

10.3 Composite Functions Part 1

Composite Function: The composition of $f(x)$ and $g(x)$ is defined as $f(g(x))$, where $g(x)$ is substituted into the equation of $f(x)$

$$f(g(x)) = (f \circ g)(x)$$

$$g(f(x)) = (g \circ f)(x)$$

Ex. #1 Given $f(x) = 3x + 2$ and $g(x) = x^2 + 1$ find the following:

a) $f(g(2))$

$$g(2) = 2^2 + 1$$

$$g(2) = 5$$

$$f(g(2)) = f(5)$$

$$= 3(5) + 2$$

$$= 15 + 2$$

$$= 17$$

c) $f(f(-2))$

$$f(-2) = 3(-2) + 2$$

$$f(-2) = -4$$

$$f(f(-2)) = f(-4)$$

$$= 3(-4) + 2$$

$$= -10$$

b) $(f \circ g)(-3)$

$$f(g(-3))$$

$$g(-3) = (-3)^2 + 1$$

$$g(-3) = 10$$

$$f(g(-3)) = f(10)$$

$$= 3(10) + 2$$

$$= 32$$

d) $(g \circ g)(2)$

$$g(g(2))$$

$$g(2) = (2)^2 + 1$$

$$g(2) = 5$$

$$g(g(2)) = g(5)$$

$$= 5^2 + 1$$

$$= 26$$

Ex. #2: Given $f(x) = 2x^2 + x$ and $g(x) = 3x + 1$, determine the equation for $f(g(x))$.

$$\begin{aligned} f(g(x)) &= 18x^2 + 12x + 2 + 3x + 1 \\ &= 18x^2 + 15x + 3 \\ f(g(x)) &= 2(3x+1)^2 + (3x+1) \\ &= 2(9x^2 + 6x + 1) + 3x + 1 \end{aligned}$$

Use $f(g(x))$ to find the following:

a) $f(g(1))$.

$$\begin{aligned} f(g(x)) &= 18x^2 + 15x + 3 \\ f(g(1)) &= 18(1)^2 + 15(1) + 3 \\ &= 18 + 15 + 3 \\ &= 36 \end{aligned}$$

b) $f(g(-2))$.

$$\begin{aligned} f(g(-2)) &= 18(-2)^2 + 15(-2) + 3 \\ &= 18(4) - 30 + 3 \\ &= 72 - 30 + 3 \\ &= 45 \end{aligned}$$

Which method is best for evaluating composite functions? (Method 1 from example #1 or method 2 from example #2) Why?

Depends on the question.

If there is only one question then method #1 may be better.

If we need to find $f(g(2))$, $f(g(-1))$, ... then use method #2

Ex. #3: Given $f(x) = -2x + 3$ and $g(x) = -x^2 + 2$, find the following:

$$\begin{aligned} \text{a) } f(f(x)) &= f(-2x+3) \\ &= -2(-2x+3)+3 \\ &= 4x-6+3 \\ &= 4x-3 \end{aligned}$$

$$\begin{aligned} \text{b) } (f \circ g)(x) &= f(g(x)) \\ &= f(-x^2+2) \\ &= -2(-x^2+2)+3 \\ &= 2x^2-4+3 \\ &= 2x^2-1 \end{aligned}$$

$$\begin{aligned} \text{c) } (g \circ f)(x) &= g(f(x)) \\ &= g(-2x+3) \\ &= -(-2x+3)^2+2 \\ &= -[4x^2-12x+9]+2 \\ &= -4x^2+12x-9+2 \\ &= -4x^2+12x-7 \end{aligned}$$

$$\begin{aligned} \text{d) } g(g(x)) &= g(-x^2+2) \\ &= -(-x^2+2)^2+2 \\ &= -[x^4-4x^2+4]+2 \\ &= -x^4+4x^2-4+2 \\ &= -x^4+4x^2-2 \end{aligned}$$