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10.3 Composite Functions Part 2

Ex. #1 Given $f(x) = \sqrt{x-4}$ and $g(x) = x^2$ find the following composite functions and any restrictions on the domain.

a) $f(g(x))$

$$f(g(x)) = f(x^2)$$

$$f(g(x)) = \sqrt{x^2 - 4}$$

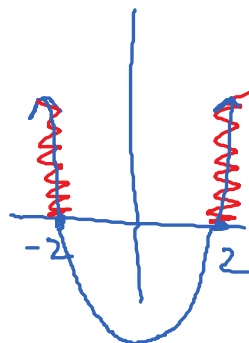
$$x^2 - 4 \geq 0$$

$$(x-2)(x+2) \geq 0$$

$$x \geq 2 \quad x \leq -2$$

or

$$|x| \geq 2$$



b) $g(f(x))$

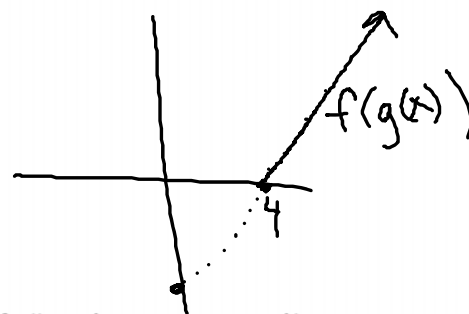
$$g(f(x)) = g(\sqrt{x-4})$$

$$g(f(x)) = (\sqrt{x-4})^2$$

$$g(f(x)) = x-4$$

$$x-4 \geq 0$$

$$x \geq 4$$



Ex. #2: Given $f(x) = \frac{1}{x-2}$ and $g(x) = 3-x$ find the following composite functions and any restrictions on the domain.

a) $f(g(x))$

$$f(g(x)) = f(3-x)$$

$$f(g(x)) = \frac{1}{(3-x)-2}$$

$$f(g(x)) = \frac{1}{-x+1}$$

$$x \neq 1$$

b) $g(f(x))$

$$g(f(x)) = g\left(\frac{1}{x-2}\right)$$

$$g(f(x)) = 3 - \frac{1}{x-2}$$

$$g(f(x)) = \frac{3(x-2)}{x-2} - \frac{1}{x-2}$$

$$g(f(x)) = \frac{3x-6-1}{x-2}$$

$$g(f(x)) = \frac{3x-7}{x-2} \quad x \neq 2$$

Ex. #3: Given $f(x) = 2x - 1$ and $g(x) = \sin x$, find $f(g(x))$ and graph the composite function. Domain and Range

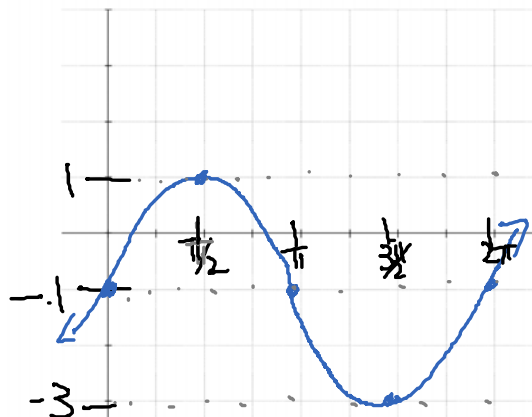
$$f(g(x)) = f(\sin x)$$

$$f(g(x)) = 2(\sin x) - 1$$

$$f(g(x)) = 2\sin x - 1$$

$$\text{Max} = 2 - 1 = 1$$

$$\text{Min} = -2 - 1 = -3$$



$$\text{Domain } \{x \mid x \in \mathbb{R}\}$$

$$\text{Range } \{y \mid -3 \leq y \leq 1, y \in \mathbb{R}\}$$

Ex. #4: Given $f(x) = 3 \cos x$ and $g(x) = 2x$, find $f(g(x))$ and graph the composite function.

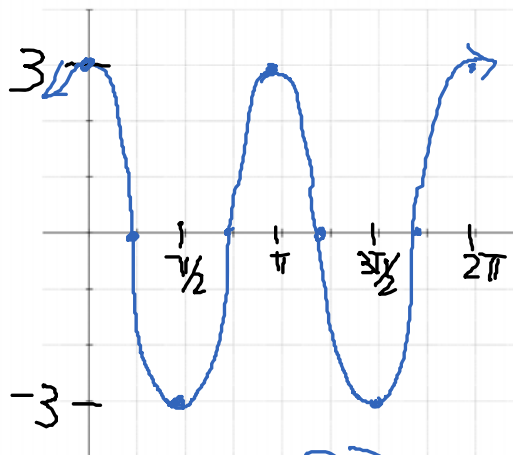
$$f(g(x)) = f(2x)$$

$$f(g(x)) = 3 \cos(2x)$$

$$f(g(x)) = 3 \cos 2x$$

$$\text{period} = \frac{2\pi}{b}$$

$$\text{period} = \frac{2\pi}{2} = \pi$$



$$\text{Domain } \{x \mid x \in \mathbb{R}\}$$

$$\text{Range } \{y \mid -3 \leq y \leq 3, y \in \mathbb{R}\}$$

If two functions are inverses of each other then $f(g(x)) = x$ and $g(f(x)) = x$

Ex. #5: Verify that $f(x) = 3^{x+1}$ and $g(x) = \log_3 x - 1$ are inverses of each other.

$$\begin{aligned} f(g(x)) &= f(\log_3 x - 1) \\ f(g(x)) &= 3^{\log_3 x - 1 + 1} \\ f(g(x)) &= 3^{\log_3 x} \\ f(g(x)) &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(3^{x+1}) \\ g(f(x)) &= \log_3 3^{x+1} - 1 \\ g(f(x)) &= (x+1)\log_3 3 - 1 \\ g(f(x)) &= (x+1)(1) - 1 \\ g(f(x)) &= x+1 - 1 \\ g(f(x)) &= x \end{aligned}$$

Ex. #6: Given $f(x) = |x + 3|$ and $g(x) = 2x$, find $f(g(x))$ and graph the composite function.

$$\begin{aligned} f(g(x)) &= f(2x) \\ f(g(x)) &= |2x + 3| \end{aligned}$$

$$\begin{aligned} y &= 2x + 3 \\ 0 &= 2x + 3 \\ -3 &= 2x \\ -\frac{3}{2} &= x \end{aligned}$$

$$\{x \mid x \in \mathbb{R}\}$$

$$\{y \mid y \geq 0, y \in \mathbb{R}\}$$

