12.2 Geometric Series

$$S_n = t_1 + t_1r^1 + t_1r^2 + t_1r^3 + \dots + t_1r^{n-1}$$

$$r(S_n) = t_1r + t_1r^2 + t_1r^3 + t_1r^4 + \dots + t_1r^n$$

subtract $rS_n - S_n$

$$\begin{array}{rl} rS_n = & (t_1r + t_1r^2 + t_1r^3 \dots \quad + t_1r^n) \\ - \; S_n = & -(t_1 + t_1r + t_1r^2 + \dots \quad + t_1r^{n-1}) \end{array}$$

$$rS_n - S_n = -t_1 + t_1r^n$$

$$S_n(r-1) = t_1(-1 + r^n)$$

$$S_n = \frac{t_1(-1+r^n)}{r-1}$$

$$\boxed{S_n = \frac{t_1(r^n - 1)}{r-1}}$$

#1 Find the sum of the first 8 terms of the series.

5 + 10 + 20 + ...

$n = 8$

$t_1 = 5$

$r = \frac{10}{5} = 2$

$S_8 = ?$

$$S_n = \frac{t_1(r^n-1)}{r-1}$$

$$S_8 = \frac{5(2^8-1)}{2-1}$$

$$S_8 = 5(256-1)$$

$S_8 = ?$

$$S_8 = \frac{5(256-1)}{1}$$

$$S_8 = 5(255)$$

$$S_8 = 1275$$

#2 Find the sum

$$8 + 4 + 2 + \ldots\ldots + \frac{1}{64}$$

$r = \frac{4}{8} = \frac{1}{2}$

$t_1 = 8$

$n = ?$ $n = 10$

$S_n = ?$ S_{10}

Find n using $t_n = t_1 r^{n-1}$

$$\frac{1}{64} = 8\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$$

$$\left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{n-1}$$

$$9 = n - 1$$

$$10 = n$$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{8\left(\left(\frac{1}{2}\right)^{10} - 1\right)}{\frac{1}{2} - 1}$$

$$S_{10} = \frac{8\left(\frac{1}{1024} - 1\right)}{-\frac{1}{2}}$$

$$S_{10} = \frac{8\left(-\frac{1023}{1024}\right)}{-\frac{1}{2}}$$

$$S_{10} = 8\left(-\frac{1023}{1024}\right)\left(-\frac{2}{1}\right)$$

$$S_{10} = 2^3\left(\frac{1023}{2^{10}}\right)(2^1)$$

$$\left(\frac{\quad}{2^{10}}\right)$$

$$S_{10} = 2^4\left(\frac{1023}{2^{10}}\right)$$

$$S_{10} = \frac{1023}{2^6}$$

$$S_{10} = \frac{1023}{64}$$

#3 $S_n = 2186$ $t_n = 1458$ $r = 3$

Find t_1 of the sequence

$$S_n = \frac{t_1(r^n - 1)}{r - 1} \qquad t_n = t_1(r)^{n-1}$$

$$2186 = \frac{t_1(3^n - 1)}{3 - 1} \qquad 1458 = t_1(3)^{n-1}$$

$$4372 = t_1(3^n - 1)$$

$$\frac{4372}{1458} = \frac{\cancel{t_1}(3^n - 1)}{\cancel{t_1}(3)^{n-1}}$$

$$\frac{4372}{1458} = \frac{3^n - 1}{3^{n-1}}$$

$$\frac{4372}{1458} = \frac{3^n}{3^{n-1}} - \frac{1}{3^{n-1}}$$

$n - (n-1)$

$n - n + 1 = 1$

$$\frac{4372}{1458} = 3^1 - 3^{-(n-1)}$$

$$\frac{4372}{1458} - 3 = -3^{-n+1}$$

$$-\frac{4372}{1458}+3=3^{-n+1}$$

$$\log\left(-\frac{4372}{1458}+3\right)=\log 3^{-n+1}$$

$$\log\left(-\frac{4372}{1458}+3\right)=(-n+1)\log 3$$

$$\log\left(-\frac{4372}{1458}+3\right)=-n\log 3+\log 3$$

$$\log\left(-\frac{4372}{1458}+3\right)-\log 3=-n\log 3$$

$$\frac{\log\left(-\frac{4372}{1458}+3\right)-\log 3}{-\log 3}=n$$

$$n=7$$

$$t_7=t_1(r)^{7-1}$$

$$1458=t_1(3)^6$$

$$\frac{1458}{3^6}=t_1$$

$$t_1=2$$

$$S_n=\frac{rt_n-t_1}{r-1}$$

#3 $S_n=2186 \quad t_n=1458 \quad r=3$

$$S_n = \frac{r t_n - t_1}{r-1}$$

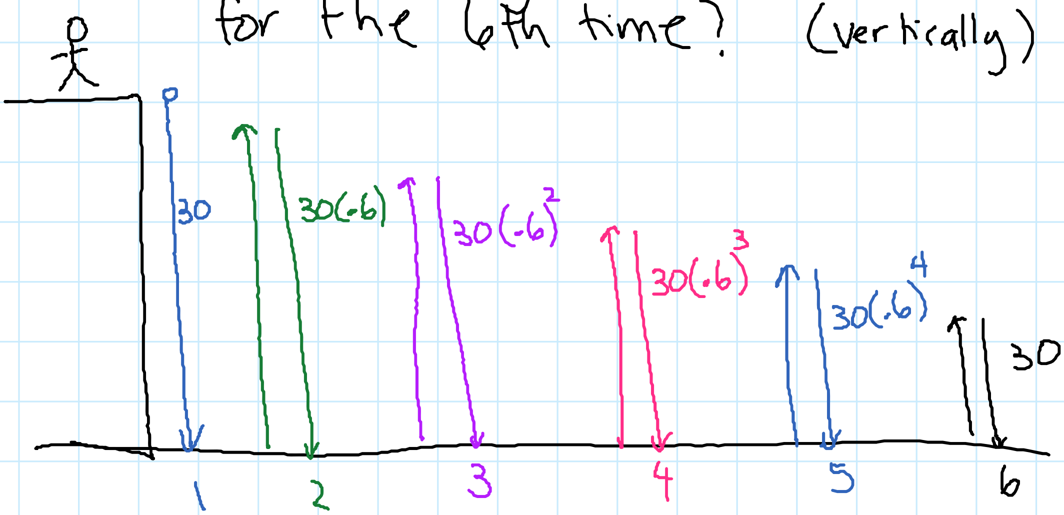
$$2186 = \frac{3(1458) - t_1}{3-1}$$

$$2186 = \frac{3(1458) - t_1}{2}$$

$$4372 = 4374 - t_1$$

$$2 = t_1$$

#4 A ball is dropped from a building that is 30m tall. It bounces back up 60% of its height. How far does ball travel by the time it hits the ground for the 6th time? (vertically)



$$\text{distance} = 30 + 2(30)(.6) + 2(30)(.6)^2 + 2(30)(.6)^3 + 2(30)(.6)^4 + 2(30)(.6)^5$$

use 60 as t_1 then subtract 30 from the sum

$$\text{distance} = S_6 - 30 \qquad t_1 = 60$$

$$r = .6$$

-30 $11-6$

$.6-1$

distance $= 113.0016$ m