12.3 Infinite geometric Series

Activity.

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots$

$S_5 =$ $S_6 =$ $S_{30} =$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$t_1 = 1$ $r = \frac{1}{2}$

$$S_5 = \frac{1\left(\left(\frac{1}{2}\right)^5 - 1\right)}{\frac{1}{2} - 1} \qquad S_6 = \frac{1\left(\left(\frac{1}{2}\right)^6 - 1\right)}{\frac{1}{2} - 1} \qquad S_{30} = \frac{1\left(\left(\frac{1}{2}\right)^{30} - 1\right)}{\frac{1}{2} - 1}$$

$S_5 = 1.9375$ $S_6 = 1.9688$ $S_{30} \approx 2$

If the sum continues indefinitely then the sum formula simplifies to

$$S = \frac{t_1}{1 - r}$$

$-1 < r < 1$

series converges to the sum

if $r > 1$ or $r < -1$
then the series diverges

#1 Find the sum if possible

$$3 + \frac{3}{4} + \frac{3}{16} + \dots$$

$$r = \frac{\frac{3}{4}}{3} = \frac{1}{4}$$

$$-1 < \frac{1}{4} < 1$$

The series converges.

$$t_1 = 3$$

$$S = \frac{t_1}{1-r}$$

$$S = \frac{3}{1-\frac{1}{4}}$$

$$S = \frac{3}{\frac{3}{4}}$$

$$S = \cancel{3} \cdot \frac{4}{\cancel{3}}$$

$$S = 4$$

#2 Write as a common fraction

$$1.626262\dots$$

$$1 + .626262\dots$$

$$1 + \underbrace{.62 + .0062 + .000062 \dots}$$

$$1 + \underbrace{.62 + .0062 + .000062\ldots}$$

$1 +$ Infinite Sum

$t_1 = 0.62 \quad r = \frac{.0062}{.62} = 0.01$

$$1 + \frac{0.62}{1-0.01}$$

$$1 + \frac{0.62}{0.99}$$

$$1 + \frac{62}{99}$$

$$\frac{99}{99} + \frac{62}{99}$$

$$\frac{161}{99}$$

#3 If the first term of a infinite geometric series is 6 and the sum is 9. Determine the common ratio.

$S = 9$

$t_1 = 6$

$$S = \frac{t_1}{1-r}$$

$$9 = \frac{6}{1-r}$$

$$9(1-r)=6$$
$$9-9r=6$$
$$3=9r$$
$$\frac{3}{9}=r$$
$$\frac{1}{3}=r$$

#4 Determine the value(s) if x, $x \neq 0$ such that the following infinite geometric series has a finite sum.

$$1+\frac{1}{4}x+\frac{1}{16}x^2+\ldots$$

series converges

$-1 < r < 1$

$$r=\frac{\text{2nd term}}{\text{1st term}}$$

$$r=\frac{\frac{1}{4}x}{1}=\frac{1}{4}x$$

$$-1 < \frac{1}{4}x < 1$$
$$-4 < x < 4$$