

12.4 Sigma Notation

Sigma Notation: The sum of n terms
 $t_1, t_2, t_3, \dots, t_n$

largest
number that
goes in for i

n

$$\sum$$

$$a_i = a_1 + a_2 + a_3 + \dots + a_n$$

$i=1$

Smallest number that goes in for i

#1 Write the series corresponding to the expression

$$\sum_{i=1}^4 (4i+1) = (4(1)+1) + (4(2)+1) + (4(3)+1) + (4(4)+1)$$

$$= 5 + 9 + 13 + 17$$

Smallest # for $i=1$

largest # for $i=4$

$$= 44$$

formula $4i+1$

#2 Evaluate

$$\sum_{i=3}^6 (i^2 - 2i)$$

$$\begin{aligned}
&= (3^2 - 2(3)) + (4^2 - 2(4)) + (5^2 - 2(5)) + (6^2 - 2(6)) \\
&= (9 - 6) + (16 - 8) + (25 - 10) + (36 - 12) \\
&= 3 + 8 + 15 + 24 \\
&= 50
\end{aligned}$$

#3 Evaluate

$$\begin{aligned}
&\sum_{j=2}^6 3^{j-1} \\
&= 3^{2-1} + 3^{3-1} + 3^{4-1} + 3^{5-1} + 3^{6-1} \\
&= 3 + 3^2 + 3^3 + 3^4 + 3^5
\end{aligned}$$

Geometric Series

$$\begin{aligned}
t_1 &= 3 \\
r &= \frac{3^2}{3} \\
r &= 3 \\
n &= 5
\end{aligned}$$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_5 = \frac{3(3^5 - 1)}{3 - 1}$$

$$S_5 = \frac{3(3^5 - 1)}{2}$$

$$S_5 = 363$$

#4 How many terms will be created by the series

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$$\sum_{i=2}^5 a_i$$

4 terms

$$\sum_{i=1}^8 a_i$$

8 terms

$$\sum_{i=10}^{150} a_i$$

141 terms

of terms = Upper limit - lower limit + 1

#5 Write in Sigma Notation.

$$6 + 12 + 24 + \dots + 192$$

$$t_1 = 6$$

$$r = \frac{12}{6} = 2$$

$$\text{Find } n \quad t_n = 192$$

$$t_n = t_1 (r)^{n-1}$$

$$192 = 6(2)^{n-1}$$

$$32 = 2^{n-1}$$

$$2^5 = 2^{n-1}$$

$$5 = n - 1$$

$$6 = n$$

#6 Find the sum

$$\sum_{i=1}^{\infty} 5\left(\frac{1}{2}\right)^{n-1} = 5 + \frac{5}{2} + \frac{5}{4} + \dots$$

Infinite Sum

$$S = \frac{t_1}{1-r}$$

$$t_1 = 5$$
$$r = \frac{1}{2}$$

$$S = \frac{5}{1 - \frac{1}{2}}$$

$$S = \frac{5}{\frac{1}{2}}$$

$$S = 10$$