## 2.1 Limits, Rate of Change and Tangent Lines

Rates of change are used to study relationships between two quantities.

Eg. Velocity: rate of change of position with respect to time

Population Growth: Growth rate with respect to time

Change in position = Velocity x change in time.

However velocity is not constant. When driving a car the driver may speed up or slow down during a time period. Therefore we will calculate average velocity.

Average Velocity = $\dfrac{\text{change in position}}{\text{length of time interval}}$

Change in position $= \Delta s = s(t_1) - s(t_0)$

Change in time (length of time interval) $= \Delta t = t_1 - t_0$

Average velocity $= \dfrac{\Delta s}{\Delta t} = \dfrac{s(t_1) - s(t_0)}{t_1 - t_0}$

1. A ball is dropped from a state of rest at time *t = 0*. The distance traveled after *t* seconds is $s(t) = 16t^2$ ft.

a) Compute the average velocity over the time period $[3,3.01]$

$t_0 = 3 \quad t_1 = 3.01$

$\Delta s = s(3.01) - s(3)$
$= 16(3.01)^2 - 16(3)^2$
$= .9616$

Average Velocity $= \dfrac{\Delta s}{\Delta t} = \dfrac{.9616}{.01}$
$= 96.16 \text{ ft/sec}$

$\Delta t = 3.01 - 3$
$= .01$

b) Shrink the time intervals and calculate the average velocities.

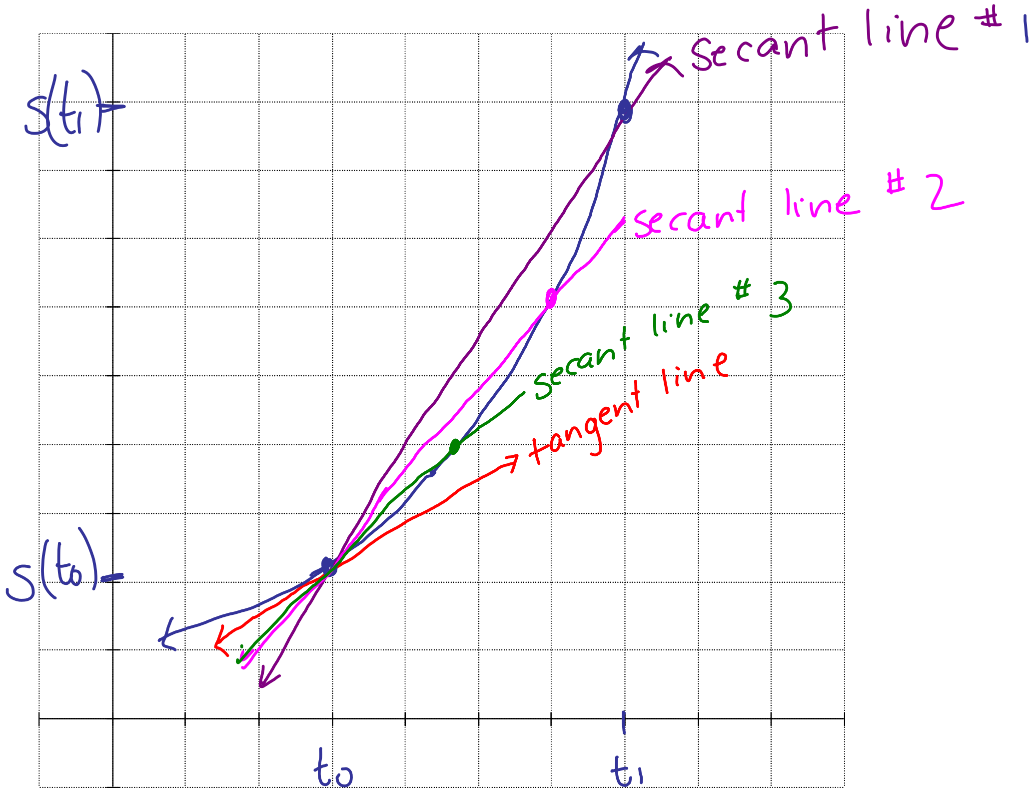
| Time Interval | Average Velocity |
|---|---|
| $[3,3.01]$ | 96.16 ft/sec |
| $[3,3.005]$ | 96.08 ft/sec |
| $[3,3.001]$ | 96.016 ft/sec |
| $[3,3.0005]$ | 96.008 ft/sec |

As the intervals shrink the average velocity is approaching 96 ft/sec

96 ft/sec is the instantaneous velocity when t=3 sec

Tangent Line: A line that touches the curve at one point

Secant Line: A line that passes through the curve at (2 points)




Slope of secant = $\dfrac{s(t_1) - s(t_0)}{t_1 - t_0}$

As the time interval shrinks the slope of the secant line approaches the slope of the tangent line

$\therefore t \to 0$

average velocity $\to$ instantaneous velocity