2.2 Limits: A Numerical and Graphical Approach

1. Investigate $\lim_{x\to 1} f(x)$ where $f(x) = \frac{x^3-1}{x-1}$ $x \neq 1$ numerically and graphically.

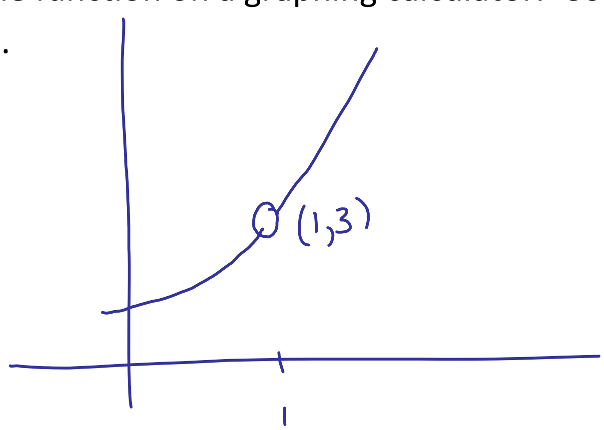
Numerically:

$x \to 1-$ From the left	$\frac{x^3-1}{x-1}$	$x \to 1+$ From the right	$\frac{x^3-1}{x-1}$
0.8	2.44	1.1	3.31
.99	2.9701	1.01	3.0301
.999	2.997	1.001	3.003

Therefore $\lim_{x\to 1} f(x) = 3$

Graphically:

Graph the function on a graphing calculator. Use the trace function to determine the limit.



Therefore $\lim_{x\to 1} f(x) = 3$

Definition of a limit:

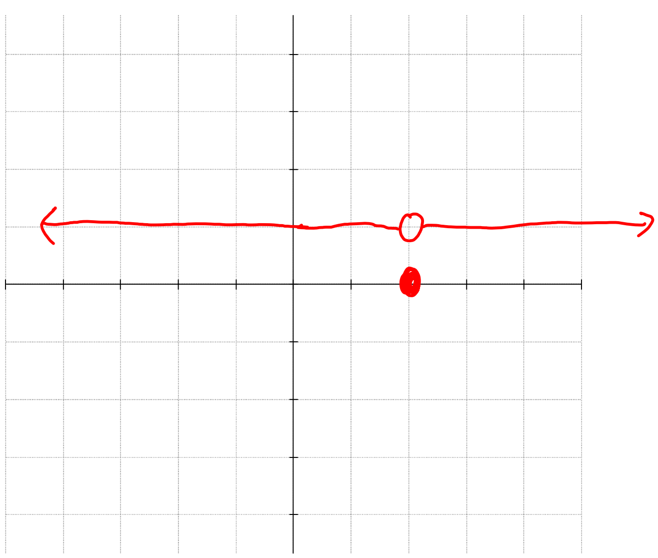
$$\lim_{x \to a} f(x) = L$$

The limit of $f(x)$ as x approaches a is equal to L

2. Find the limit numerically $\lim_{x\to 0} \frac{x}{\sqrt{x+1}-1}$ $= 2$

$x \to 0^-$ Left	$f(x)$	$x \to 0^+$ Right	$f(x)$
-0.1	1.94	0.1	2.05
-0.01	1.995	0.01	2.005
-0.001	1.9995	0.001	2.0005

3. Determine $\lim_{x\to 2} f(x)$ graphically given that $f(x) = \begin{cases} 1 & x \neq 2 \\ 0 & x = 2 \end{cases}$



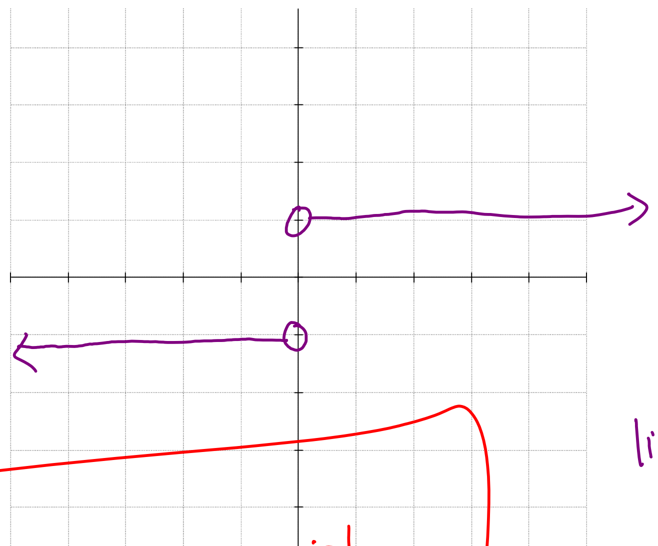
$f(2) = 0$

$\lim_{x \to 2^-} f(x) = 1$
(left)

$\lim_{x \to 2^+} f(x) = 1$
(right)

$\lim_{x \to 2} f(x) = 1$ Limit exists. Limit from left = Limit from Right

4. Determine $\lim_{x\to 0} f(x)$ graphically given that $f(x) = \frac{|x|}{x}$



$\lim_{x\to 0^-} f(x) = -1$

$\lim_{x\to 0^+} f(x) = 1$

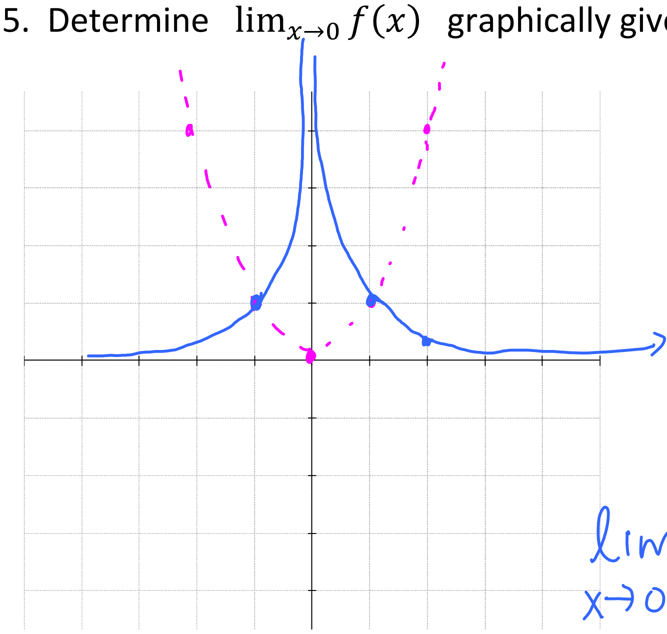
limit from the left $\neq$ limit from the right

$\therefore \lim_{x\to 0} f(x)$ does not exist.

For a limit to exist

$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$

5. Determine $\lim_{x\to 0} f(x)$ graphically given that $f(x) = \frac{1}{x^2}$



$\lim_{x\to 0^-} f(x) = \infty$

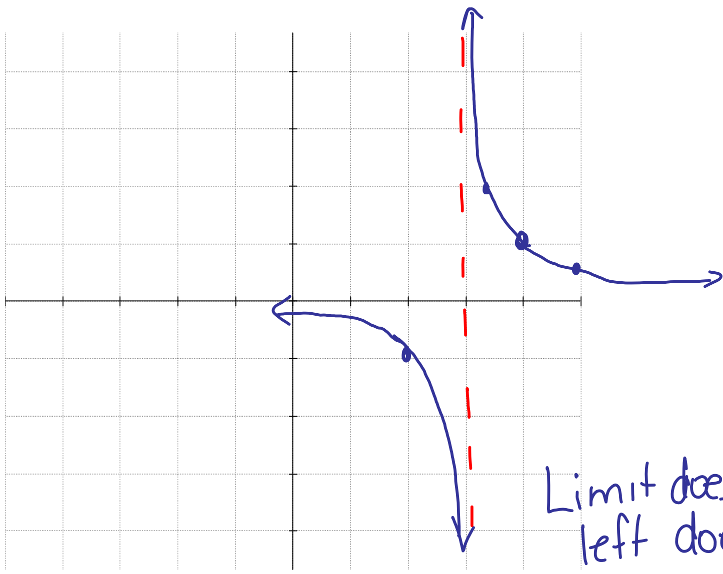
$\lim_{x\to 0^+} f(x) = \infty$

$\lim_{x\to 0} f(x)$ is not approaching a real number "L".

It is unbounded.

$\lim_{x\to 0} f(x) = \infty$

6. Determine $\lim_{x \to 3} f(x)$ graphically given that $f(x) = \frac{1}{x-3}$



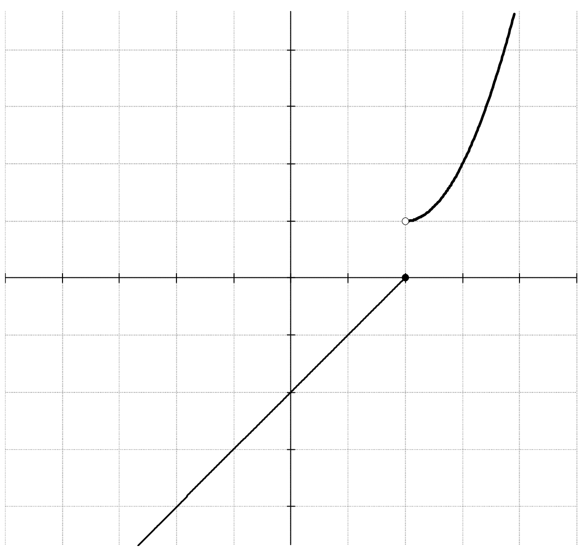
Joe average function $y = \frac{1}{x}$

$\lim_{x \to 3^-} f(x) = -\infty$

$\lim_{x \to 3^+} f(x) = \infty$

Limit does not exist. Limit from the left does not equal limit from the right.

7. Determine the limits.



a) $\lim_{x \to 0} f(x) = -2$

b) $\lim_{x \to 3} f(x) = 2$

c) $\lim_{x \to 2^-} f(x) = 0$

d) $\lim_{x \to 2^+} f(x) = 1$

e) $\lim_{x \to 2} f(x) =$ Does not exist

One sided limits exist but not $\lim_{x \to 2} f(x)$, since left limit $\neq$ right limit

$\lim_{x \to c} x = c$ $\lim_{x \to c} k = k$