2.3 Basic Limit Laws

For any constants *k* and *c*,

$$\lim_{x\to c} k = k \qquad \lim_{x\to c} x = c$$

1. $\lim_{x\to 2} 3 =$ 3

2. $\lim_{x\to 5} x =$ 5

If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist, then

- Sum Law $\lim_{x\to c}(f(x)+g(x)) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$
- Constant Multiple Law
  K is a constant $\lim_{x\to c} Kf(x) = K\cdot \lim_{x\to c} f(x)$
- Product Law
  $\lim_{x\to c}(f(x)\cdot g(x)) = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right]$
- Quotient Law
  if $\lim_{x\to c} g(x) \neq 0$ $\quad \lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$
- Powers and Roots
  $\lim_{x\to c} [f(x)]^{p/q} = \left[\lim_{x\to c} f(x)\right]^{p/q}$
  $\lim_{x\to c} x^{p/q} = \left[\lim_{x\to c} x\right]^{p/q} = c^{p/q}$
  $q \neq 0$

3. $\lim_{x\to 3} x^2 =$

$= \left[\lim_{x\to 3} x\right]^2$

$= 3^2$

$= 9$

4. $\lim_{x\to 2}(6x^3 - 7) =$

$= \lim_{x\to 2} 6x^3 - \lim_{x\to 2} 7$

$= 6\left[\lim_{x\to 2} x^3\right] - \lim_{x\to 2} 7$

$= 6\left[\lim_{x\to 2} x\right]^3 - \lim_{x\to 2} 7$

$= 6(2)^3 - 7 \quad = 41$

5. Assuming that $\lim_{x\to6} f(x) = 4$, find

a) $\lim_{x\to6}[f(x)]^2 =$

$= \left[\lim_{x\to6} f(x)\right]^2$

$= 4^2 = 16$

b) $\lim_{x\to6} \frac{1}{f(x)} = \frac{\lim_{x\to6} 1}{\lim_{x\to6} f(x)}$

$= \frac{1}{4}$

c) $\lim_{x\to6} xf(x) =$

$= \left[\lim_{x\to6} x\right] \cdot \left[\lim_{x\to6} f(x)\right]$

$= 6 \cdot 4$

$= 24$

6. $\lim_{x\to2} \frac{x^2-3x+5}{x+3} =$

$= \frac{2^2-3(2)+5}{2+3}$

$= \frac{4-6+5}{5} = \frac{3}{5}$

7. $\lim_{x\to0} \sqrt{x^2+4} =$

$= \sqrt{0^2+4}$

$= \sqrt{4}$

$= 2$

8. $\lim_{x\to2}(x+1)(3x^2-9) =$

$= (2+1)(3(2)^2-9)$

$= (3)(12-9)$

$= (3)(3)$

$= 9$

9. $\lim_{x\to5} x^{\frac{-1}{2}}(x+3)^{\frac{2}{3}} =$

$5^{-\frac{1}{2}}(5+3)^{2/3}$

$\frac{1}{5^{1/2}} \cdot (8)^{2/3}$

$\frac{1}{\sqrt[2]{5}} \left(\sqrt[3]{8}\right)^2$

$\frac{1}{\sqrt{5}} \cdot 4$

$\frac{4}{\sqrt{5}}$

$\frac{4\sqrt{5}}{5}$

10. Assuming that $\lim_{x\to-1} f(x) = 3$ and $\lim_{x\to-1} g(x) = 4$, find

$\lim_{x\to-1} \frac{f(x)g(x)-2}{3g(x)+2} = \frac{\left[\lim_{x\to-1} f(x)\right]\left[\lim_{x\to-1} g(x)\right] - \lim_{x\to-1} 2}{3\left[\lim_{x\to-1} g(x)\right] + \lim_{x\to-1} 2}$

$= \frac{(3)(4)-2}{3(4)+2} = \frac{10}{14} = \frac{5}{7}$