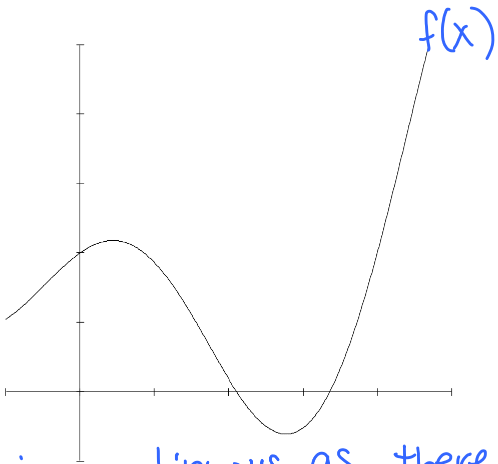
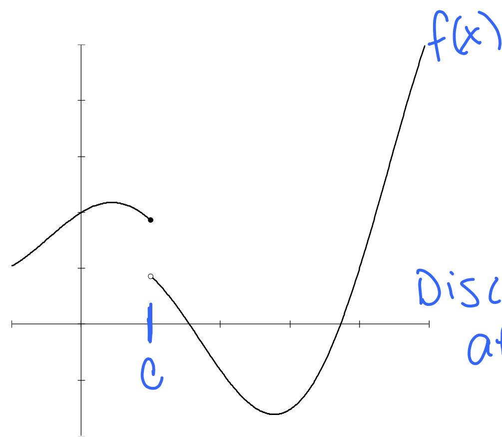
2.4 Limits and Continuity



f(x) is continuous as there are no breaks or interuptions



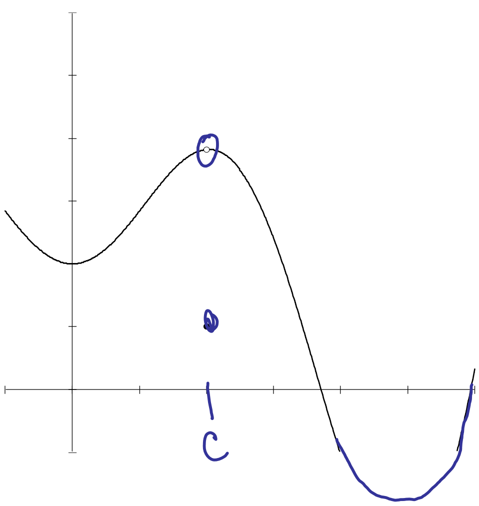
Discontinuity at x=c

Definition of Continuity:

A function *f* is continuous at *c* if the following 3 conditions are met.

1. $f(c)$ is defined
2. $\lim_{x\to c} f(x)$ must exist
3. $\lim_{x\to c} f(x) = f(c)$

Discontinuities:

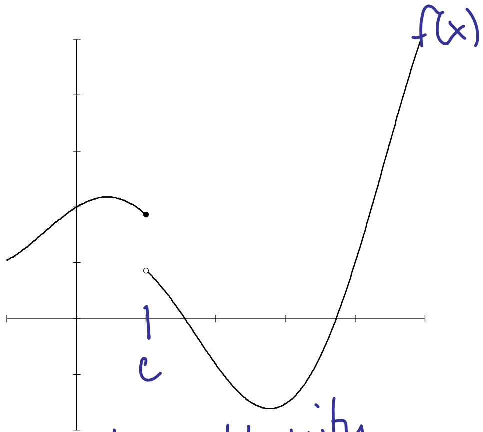


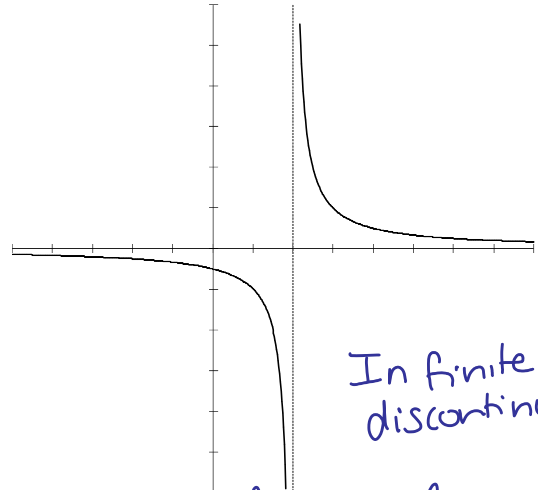
$\lim_{x\to c} f(x) \neq f(c)$

This is a removable discontinuty

We can redefine f(c) so that

$\lim_{x\to c} f(x) = f(c)$





Jump discontinuity

$\lim_{x \to c} f(x)$ does not exist $\quad$ limit left $\neq$ limit right

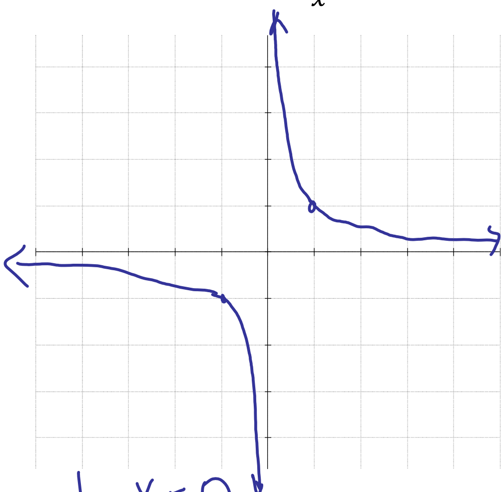
Infinite discontinuity

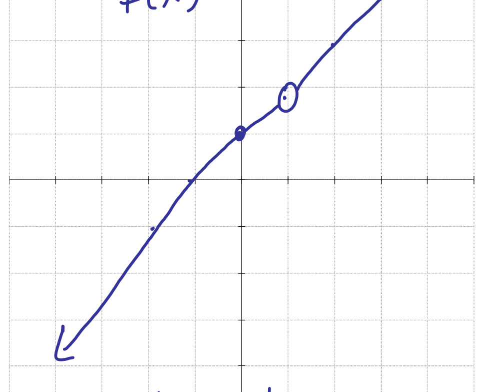
lim left or lim right $\to \infty$ or $-\infty$

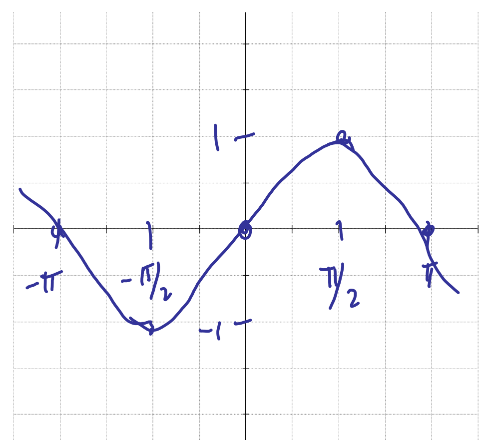
1. Determine if the following functions are continuous. If a discontinuity exists determine the type.

a) $f(x) = \frac{1}{x}$ $\qquad$ b) $f(x) = \frac{x^2-1}{x-1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$ $\qquad$ c) $f(x) = \sin x$

$f(x) = x+1$







at $x=0$
Infinite discontinuity

at $x=1$
removable discontinuity

continuous.

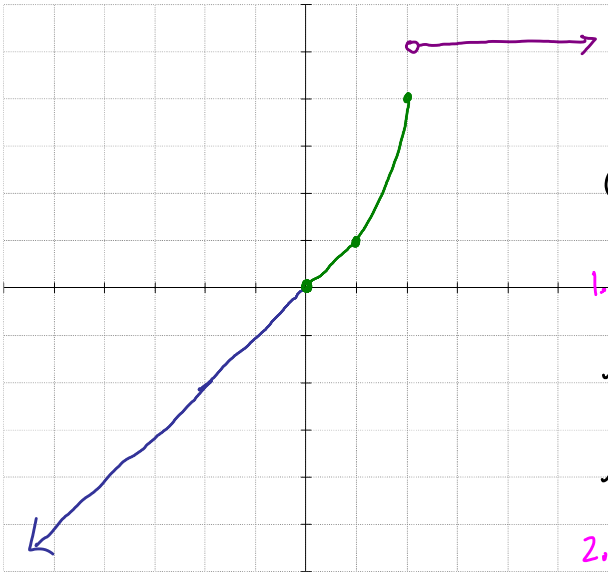
Definition: **One Sided Continuity**

A function $f(x)$ is called

Left continuous at $x = c$ if $\lim_{x \to c^-} f(x) = f(c)$

Right continuous at $x = c$ if $\lim_{x \to c^+} f(x) = f(c)$

2. Discuss the continuity of the function. $F(x) = \begin{cases} x & x < 0 \\ x^2, & 0 \le x \le 2 \\ 5 & x > 2 \end{cases}$



possible discontinuities $x=0, x=2$

consider $x=0$

1. $f(0)=0$

$\lim_{x \to 0^-} f(x) = 0$

$\lim_{x \to 0^+} f(x) = 0$

2. Limit exists

3. $f(0) = \lim_{x \to 0} f(x)$

continuous at $x=0$

consider $x=2$

1. $f(2)=4$

2. $\lim_{x \to 2^-} f(x) = 4$

$\lim_{x \to 2^+} f(x) = 5$

limit does not exist

$\therefore$ Jump discontinuity at $x=2$

Laws of Continuity: If $f(x)$ and $g(x)$ are continuous at $x=c$ then the following functions are also continuous.

From the basic limit laws

$f(x)+g(x)$ and $f(x)-g(x)$

$Kf(x)$ for any constant K

$f(x) \cdot g(x)$

$\frac{f(x)}{g(x)}$ if $g(c) \neq 0$

Continuity of Polynomial and Rational Functions: Let $P(x)$ and $Q(x)$ be polynomials.

$P(x)$ is continuous

$\frac{P(x)}{Q(x)}$ is continuous for all $x=c$ if $Q(c) \neq 0$

Continuity of some basic function:

$y = x^{\frac{1}{n}}$ $n \in I$

$y = x^{1/2}$

n is even

continuous

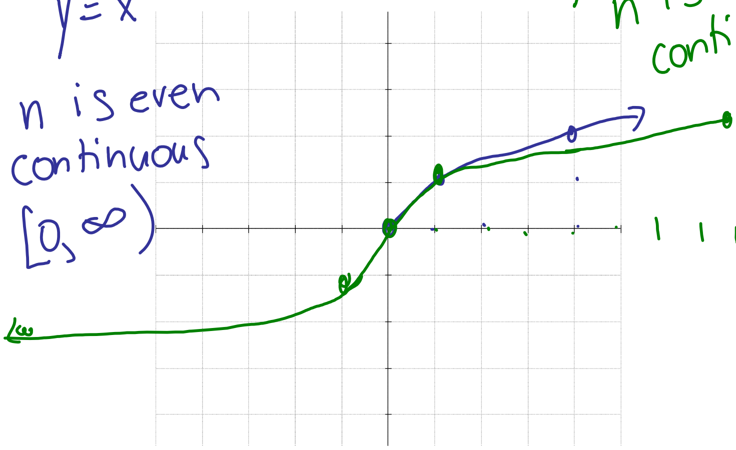
$[0, \infty)$

$y = x^{\frac{1}{3}}$

n is odd

continuous

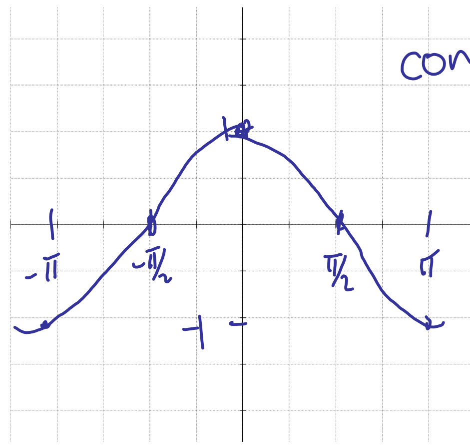
$(-\infty, \infty)$



$y = \sin x$ $\boxed{y = \cos x}$

continuous

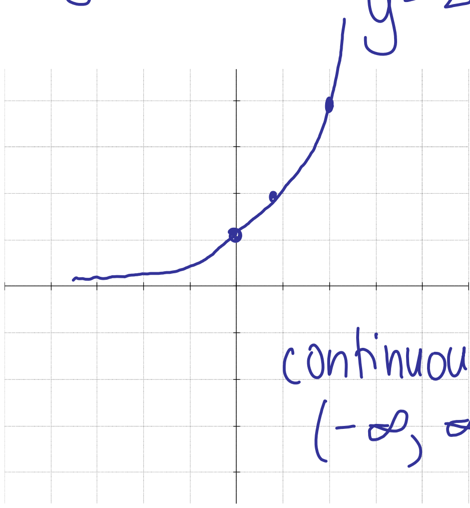
$(-\infty, \infty)$



$y = b^x$

$b > 0$ $b \neq 1$

$y = 2^x$



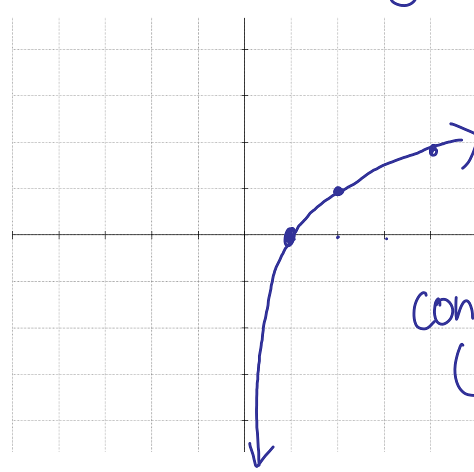
continuous

$(-\infty, \infty)$

$y = \log_b x$

$b > 0$ $b \neq 1$

$y = \log_2 x$



continuous

$(0, \infty)$

Continuity of Composite Funcions: If *g* is continuious at *x=c*, and if f is continuous at *x=g(c)*, then

$F(x) = f(g(x))$ is continuous at $x = c$

AP Calculus

Substitution method for evaluating limits:

$$\lim_{x \to c} f(x) = f(c)$$

When the function $f(x)$ is continuous at $x=c$

3. Evaluate $\lim_{x \to -1} \frac{2^x}{\sqrt{x+5}}$ if possible

Numerator 2^x continuous at $x=-1$

Denominator $\sqrt{x+5}$ continuous at $x=-1$

$\therefore \frac{2^x}{\sqrt{x+5}}$ is continuous at $x=-1$

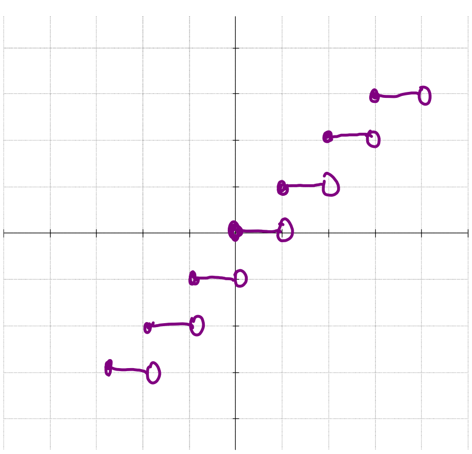
$$\lim_{x \to -1} \frac{2^x}{\sqrt{x+5}} = \frac{2^{-1}}{\sqrt{-1+5}}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{4}}$$

$$= \frac{1}{2} \cdot \frac{1}{2}$$

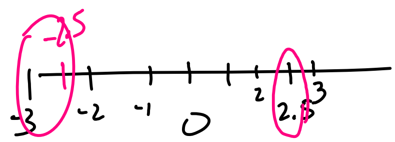
$$= \frac{1}{4}$$

4. Evaluate $\lim_{x \to 1} [\![x]\!]$ if possible. $[\![x]\!]$= greatest integer funcion, n $\quad n \leq x$



$[\![2.5]\!] = 2$

$[\![-2.5]\!] = -3$



at $x=1$ $\quad f(1)=1$

$$\lim_{x \to 1^-} f(x) = 0 \neq \lim_{x \to 1^+} f(x) = 1$$

Limit does not exist

However the one sided limits do exist