2.5 Limits Algebraically

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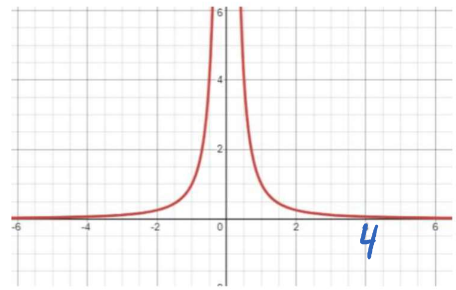
2.5 Evaluating Limits Algebraically

Substitution can be used to evaluate limits when the function is continuous

$$\lim_{x\to 4} x^{-2} = 4^{-2} = \frac{1}{16}$$



However, when $f(c)$ is not defined substitution cannot by used directly. We will need to rewrite the function algebraically.

direct sub

$$\frac{4^2-16}{4-4} = \frac{0}{0}$$

#1 $\lim_{x\to 4} \frac{x^2-16}{x-4}$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}(x+4)}{\cancel{x-4}}$$

$$= \lim_{x\to 4} x+4$$

$$= 4+4$$

$$= 8$$

Indeterminate Form

At $x = c$ if $f(c)$ yields:

$$\frac{0}{0}, \frac{\infty}{\infty}, \infty\cdot 0, \infty - \infty$$

$$\boxed{1^{\infty}, \infty^{0}, 0^{0}}$$ ch 7

AP Calculus

#2 $\lim_{x\to 2}\frac{x^3-8}{x^2-3x+2}$

direct sub

$$\frac{2^3-8}{2^2-3(2)+2} = \frac{0}{0}$$

$(x-2)$ is a factor of x^3-8

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & -8 \\ & & -2 & -4 & -8 \\ \hline & \boxed{1 \quad 2 \quad 4} & & & 0 \end{array}$$

$0-(-2)$

$0-(-4)$

$-8-(-8)$

$$\lim_{x\to 2}\frac{\cancel{(x-2)}(1x^2+2x+4)}{(x-1)\cancel{(x-2)}}$$

$$\lim_{x\to 2}\frac{x^2+2x+4}{x-1}$$

$$= \frac{2^2+2(2)+4}{2-1}$$

$$= \frac{4+4+4}{1} = \frac{12}{1} = 12$$

#3 $\lim_{h\to 4}\frac{(h+2)^2-9h}{h-4}$

direct sub

$$\frac{(4+2)^2-9(4)}{4-4} = \frac{36-36}{0} = \frac{0}{0}$$

$$= \lim_{h\to 4}\frac{h^2+4h+4-9h}{h-4}$$

$$= \lim_{h\to 4}\frac{h^2-5h+4}{h-4}$$

$$= \lim_{h\to 4}\frac{\cancel{(h-4)}(h-1)}{\cancel{h-4}}$$

$$= \lim_{h\to 4} h-1$$

$$= 4-1$$

$$= 3$$

$$\frac{1}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

AP Calculus

#4 $\lim\limits_{x\to 0}\frac{\sqrt{x+1}-1}{x}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$

direct sub

$$\frac{\sqrt{0+1}-1}{0}=\frac{0}{0}$$

$$=\lim_{x\to 0}\frac{x+1+\sqrt{x+1}-\sqrt{x+1}-1}{x(\sqrt{x+1}+1)}$$

$$=\lim_{x\to 0}\frac{x+1-1}{x(\sqrt{x+1}+1)}$$

$$=\lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}(\sqrt{x+1}+1)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{x+1}+1}$$

$$=\frac{1}{\sqrt{0+1}+1}=\frac{1}{2}$$

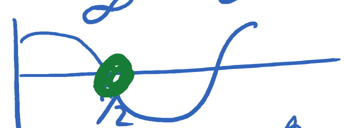
#5 $\lim\limits_{x\to\frac{\pi}{2}}(\sec x-\tan x)$

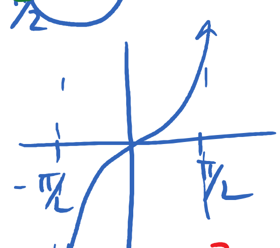
$$\sec\frac{\pi}{2}-\tan\pi/2$$

$$\frac{1}{\cos\pi/2}-\tan\pi/2$$

$$\frac{1}{0}-\infty$$

$$\infty-\infty$$





$$=\lim_{x\to\frac{\pi}{2}}\frac{1}{\cos x}-\frac{\sin x}{\cos x}$$

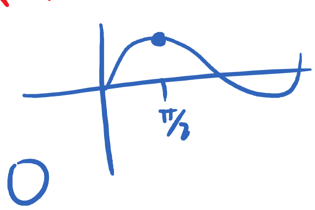
$$=\lim_{x\to\pi/2}\frac{1-\sin x}{\cos x}\cdot\frac{1+\sin x}{1+\sin x}$$

$$=\lim_{x\to\pi/2}\frac{1+\sin x-\sin x-\sin^2x}{\cos x(1+\sin x)}$$

$$=\lim_{x\to\pi/2}\frac{1-\sin^2x}{\cos x(1+\sin x)}$$

$\sin^2x+\cos^2x=1$

$$=\lim_{x\to\pi/2}\frac{\cos^{\cancel{2}}x}{\cancel{\cos x}(1+\sin x)}$$



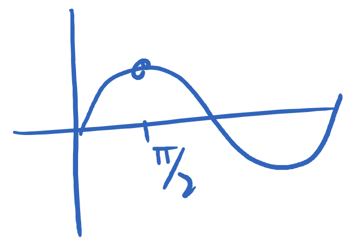
$$=\lim_{x\to\pi/2}\frac{\cos x}{1+\sin x}=\frac{\cos\pi/2}{1+\sin\pi/2}=\frac{0}{1+1}=0$$

AP Calculus

#6 $\lim_{x\to\frac{\pi}{2}} \frac{\tan x}{\sec x}$

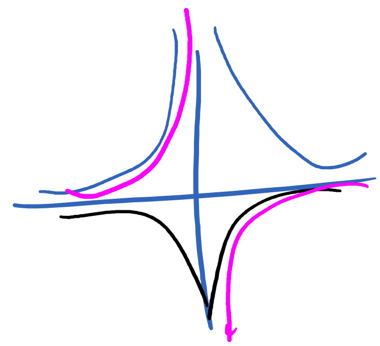
$$\frac{\tan \pi/2}{\sec \pi/2} = \frac{\infty}{\frac{1}{0}} = \frac{\infty}{\infty}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}}$$



$$\lim_{x\to\frac{\pi}{2}} \frac{\sin x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{1}$$

$$\lim_{x\to\pi/2} \sin x = \sin \pi/2 = 1$$



#7 $\lim_{x\to 2} \frac{x^2-x+5}{x-2}$

$$\begin{array}{r|l} & x+1 \\ \hline x-2 & x^2-x+5 \\ & x^2-2x \\ \hline & 1x+5 \\ & x-2 \\ \hline & 7 \end{array}$$

$-x-(-2x)$

$5-(-2)$

$$\lim_{x\to 2}\left[\frac{7}{x-2} + x+1\right]$$

$$= \lim_{x\to 2} \frac{7}{x-2} + \lim_{x\to 2}(x+1)$$

$= DNE + (2+1)$

$= DNE + 3$

$= DNE$

$y = \frac{7}{x-2}$