2.6 Factoring Special Polynomial

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A. Factoring PERFECT SQUARE TRINOMIALS

A perfect square trinomial has the form: $ax^2 + bx + c$

Perfect square trinomials have the following characteristics:

- a and c must be perfect squares (1, 4, 9, 16, etc....)
- bx is equal to twice the product of the square roots of terms a and c.

$$bx = 2\left(\sqrt{ax^2}\right)\left(\sqrt{c}\right)$$

Example 1: Verify that the following trinomials are perfect square trinomials.

a) $4x^2 + 12x + 9$

$4x^2$ and 9 is a perfect square

$\sqrt{4x^2} = 2x$ $\sqrt{9} = 3$

$2(2x)(3) = 12x$

Perfect square trinomial

b) $9x^2 - 6x + 1$

$9x^2$ and 1 are perfect squares

$\sqrt{9x^2} = 3x$ $\sqrt{1} = 1$

$2(3x)(1) = 6x$

yes perfect square

Use decomposition to factor the polynomials

a) $4x^2 + 12x + 9$ $\quad 4(9)$ $\quad 6 \times 6 = 36$ $\quad 6 + 6 = 12$

$4x^2 + 6x + 6x + 9$

$2x(2x+3) + 3(2x+3)$

$(2x+3)(2x+3)$

$(2x+3)^2$

b) $9x^2 - 6x + 1$ $\quad 9(1)$ $\quad -3 \times -3 = 9$ $\quad -3 + -3 = -6$

$9x^2 - 3x - 3x + 1$

$3x(3x-1) - 1(3x-1)$

$(3x-1)(3x-1)$

$(3x-1)^2$

Find a pattern for factoring perfect ~~square~~ trinomials:

$$(a+b)^2 = (a+b)(a+b)$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

Example 2: Factor the following trinomials

a) $4x^2 + 4x + 1$

$\sqrt{4x^2} = 2x \quad \sqrt{1} = 1$

$2(2x)(1) = 4x$

$(2x + 1)^2$

b) $4x^2 - 12x + 9$

$\sqrt{4x^2} = 2x \quad \sqrt{9} = 3$

$2(2x)(3) = 12x$

$(2x - 3)^2$

c) $4 - 20x + 25x^2$

$\sqrt{4} = 2 \quad \sqrt{25x^2} = 5x$

$2(2)(5x) = 20x$

$(2 - 5x)^2$

d) $2x^2 - 4x + 2$

$2(x^2 - 2x + 1)$

$\sqrt{x^2} = x \quad \sqrt{1} = 1$

$2(x)(1) = 2x$

$2(x - 1)^2$

B. Factoring the DIFFERENCE OF SQUARES

The difference of squares has the form: $a^2 - b^2$

The difference of squares has the following characteristics:

- There are only two terms in the polynomial (binomial)
- Each term is a perfect square
- Second term must be subtracted from the first term

Observe the pattern when expanding two binomials with opposite signs

$(a+b)(a-b) = a^2 - ab + ab - b^2$

$= a^2 - b^2$

Example 3: Factor the following binomials

a) $16x^2 - 25$

$\sqrt{16x^2} = 4x \quad \sqrt{25} = 5$

$(4x+5)(4x-5)$

b) $49x^2 - 121$

$\sqrt{49x^2} = 7x$

$\sqrt{121} = 11$

$(7x+11)(7x-11)$

c) $3x^3 - 147x$

$3x(x^2 - 49)$

$\sqrt{x^2} = x \quad \sqrt{49} = 7$

$3x(x+7)(x-7)$