2.7 Limits at Infinity

The notation $x \to \infty$ indicates that x increases without bound.

The notation $x \to -\infty$ indicates that x decreases (through negative values) without bound.

$\lim_{x \to \infty} f(x) = L$ if f(x) gets closer to L as $x \to \infty$

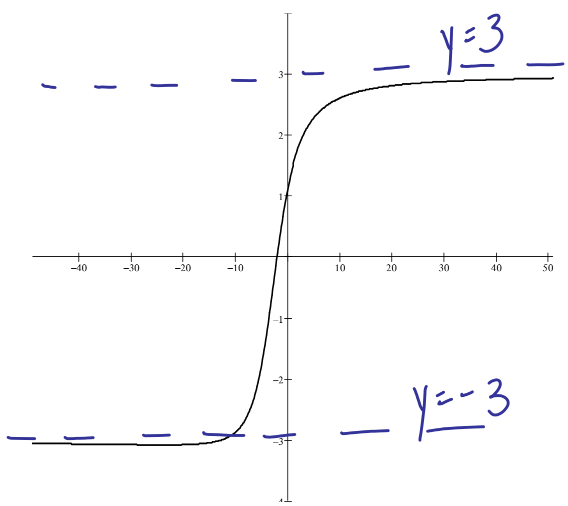
and

$\lim_{x \to -\infty} f(x) = L$ if f(x) gets closer to L as $x \to -\infty$

Horizontal asymptote at $y = L$

1. Discuss the asymptotic behavior of the graph.

determine the asymptotes



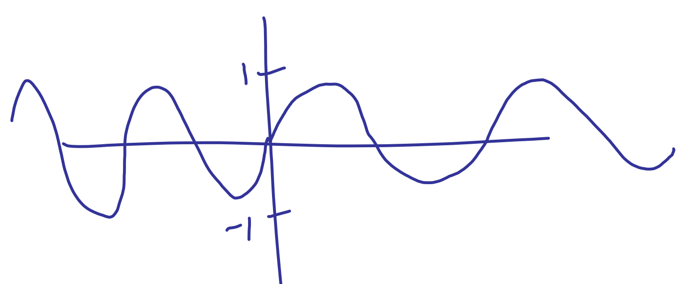
$\lim_{x \to \infty} f(x) = 3$

$\lim_{x \to -\infty} f(x) = -3$

Horizontal asymptotes at $y=3$ and $y=-3$

Limits at infinity do not always exist.

- $f(x) = \sin x$



$\left.\begin{array}{l} \lim_{x \to \infty} \sin x \\ \lim_{x \to -\infty} \sin x \end{array}\right\}$ limit does not exist

sinx does not approach a real number

Oscillating behaviour between -1 and 1

AP Calculus

- $f(x) = x^n \;\; n > 0$
  n is even

$$\lim_{x \to \infty} x^n = \infty$$

$$\lim_{x \to -\infty} x^n = \infty$$

unbounded

$f(x) = x^n \;\; n > 0$
n is odd

$$\lim_{x \to \infty} x^n = \infty$$

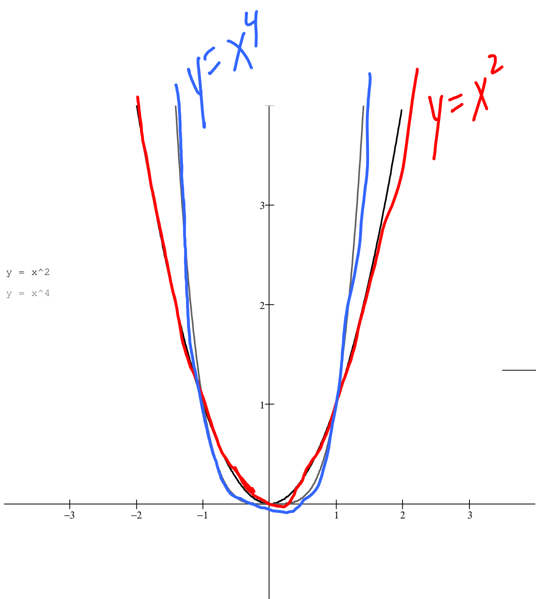
$$\lim_{x \to -\infty} x^n = -\infty$$

unbounded

$f(x) = x^{-n} \;\; n > 0$

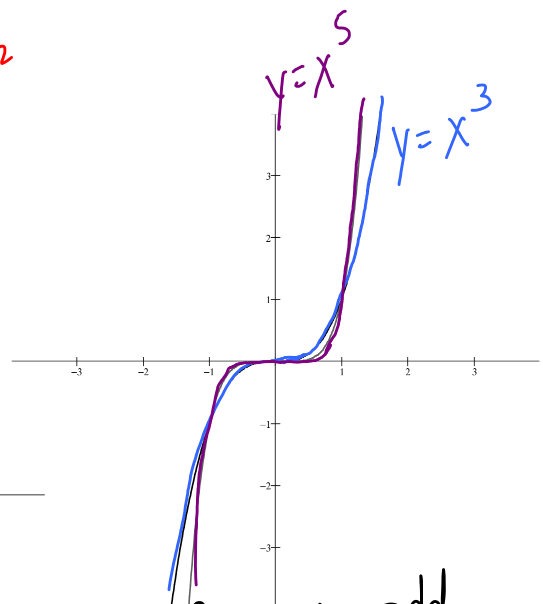
$$\lim_{x \to \infty} \frac{1}{x^n} = 0$$

$$\lim_{x \to -\infty} \frac{1}{x^n} = 0$$



if n is even

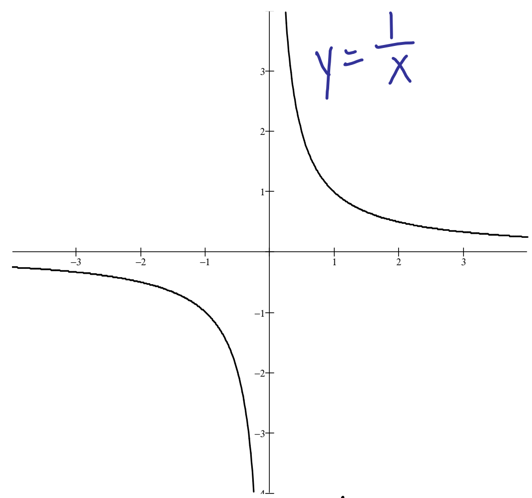
$$\lim_{x \to \pm\infty} x^n = \infty$$



if n is odd

$$\lim_{x \to \infty} x^n = \infty$$

$$\lim_{x \to -\infty} x^n = -\infty$$



$$\lim_{x \to \pm\infty} \frac{1}{x^n} = 0$$

2. Calculate $\lim_{x \to \infty} 5 - \frac{2}{x^2}$

$$\lim_{x \to \infty} 5 - \lim_{x \to \infty} \frac{2}{x^2}$$

$$\lim_{x \to \infty} 5 - 2\left[\lim_{x \to \infty} \frac{1}{x^2}\right]$$

$$5 - 2(0)$$

$$= 5$$

AP Calculus

3. Calculate $\lim_{x\to\infty} \frac{4x^2-5x+7}{10x+x^3}$

Subsitution leads to the indeterminate form.

Rewrite the question

Divide very term by the highest power in the demoninator

divide by x^3

$$\lim_{x\to\infty} \frac{\frac{4x^2}{x^3} - \frac{5x}{x^3} + \frac{7}{x^3}}{\frac{10x}{x^3} + \frac{x^3}{x^3}}$$

$$= \lim_{x\to\infty} \frac{\frac{4}{x} - \frac{5}{x^2} + \frac{7}{x^3}}{\frac{10}{x^2} + 1}$$

$$= \frac{\lim_{x\to\infty} \frac{4}{x} - \frac{5}{x^2} + \frac{7}{x^3}}{\lim_{x\to\infty} \frac{10}{x^2} + 1}$$

$$= \frac{0 - 0 + 0}{0+1} = \frac{0}{1} = 0$$

4. Calculate $\lim_{x\to\pm\infty} \frac{3x-2}{\sqrt{2x^2+1}}$

The Quotient law gives $\frac{\infty}{\infty}$ Indeterminate form

Divide by the highest power in the denominator.

$\sqrt{x^2} = x \quad x \to \infty$

$$\lim_{x\to\infty} \frac{\frac{3x}{x} - \frac{2}{x}}{\frac{\sqrt{2x^2+1}}{\sqrt{x^2}}}$$

$$= \lim_{x\to\infty} \frac{3 - \frac{2}{x}}{\sqrt{\frac{2x^2}{x^2} + \frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{3 - \frac{2}{x}}{\sqrt{2 + \frac{1}{x^2}}}$$

$$= \frac{\lim_{x\to\infty} 3 - \frac{2}{x}}{\left(\lim_{x\to\infty} 2 + \frac{1}{x^2}\right)^{1/2}}$$

$$= \frac{3-0}{(2+0)^{1/2}} = \frac{3}{\sqrt{2}}$$

AP Calculus

$$\lim_{x \to -\infty} \frac{3x - 2}{\sqrt{2x^2 + 1}}$$

$x = -\sqrt{x^2}$ $\quad x < 0$

$$\lim_{x \to -\infty} \frac{\frac{3x}{x} - \frac{2}{x}}{\frac{\sqrt{2x^2+1}}{-\sqrt{x^2}}}$$

$$\lim_{x \to -\infty} \frac{3 - \frac{2}{x}}{-\sqrt{2 + \frac{1}{x^2}}}$$

$$\frac{\lim_{x \to -\infty} 3 - \frac{2}{x}}{-\left[\lim_{x \to -\infty} \left(2 + \frac{1}{x^2}\right)\right]^{1/2}}$$

$$\frac{3}{-\sqrt{2}}$$