

AP Calculus

AB Practice Exam 2012

Part 1: A calculator may NOT be used on this part of the exam.

1. If $y = x \sin x$, then $\frac{dy}{dx} =$

- (A) $\sin x + \cos x$
- (B) $\sin x + x \cos x$
- (C) $\sin x - x \cos x$
- (D) $x(\sin x + \cos x)$
- (E) $x(\sin x - \cos x)$

2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

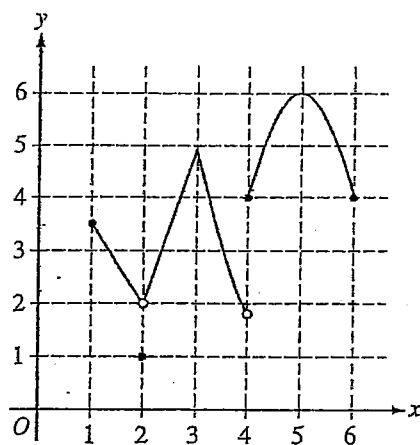
- (A) $(-\infty, -10]$ and $[10, \infty)$
- (B) $[-10, 10]$
- (C) $[0, 10]$ only
- (D) $[0, 10\sqrt{3}]$ only
- (E) $[0, \infty)$

3. $\int \sec x \tan x \, dx =$

- (A) $\sec x + C$
- (B) $\tan x + C$
- (C) $\frac{\sec^2 x}{2} + C$
- (D) $\frac{\tan^2 x}{2} + C$
- (E) $\frac{\sec^2 x \tan^2 x}{2} + C$

4. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$

- (A) 4
- (B) 5
- (C) 6
- (D) 7
- (E) 8



Graph of f

5. The graph of the function f is shown above. Which of the following statements is false?

(A) $\lim_{x \rightarrow 2} f(x)$ exists.

(B) $\lim_{x \rightarrow 3} f(x)$ exists.

(C) $\lim_{x \rightarrow 4} f(x)$ exists.

(D) $\lim_{x \rightarrow 5} f(x)$ exists.

(E) The function f is continuous at $x = 3$.

6. A particle moves along the x -axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

- (A) 3 (B) 6 (C) 9 (D) 18 (E) 27

7. If $y = (x^3 - \cos x)^5$, then $y' =$

(A) $5(x^3 - \cos x)^4$

(B) $5(3x^2 + \sin x)^4$

(C) $5(3x^2 + \sin x)$

(D) $5(3x^2 + \sin x)^4 \cdot (6x + \cos x)$

(E) $5(x^3 - \cos x)^4 \cdot (3x^2 + \sin x)$

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

- (A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

$$f(x) = \begin{cases} \frac{(2x+1)(x-2)}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

9. Let f be the function defined above. For what value of k is f continuous at $x = 2$?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 5

10. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

- (A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x . Which of the following statements is true?

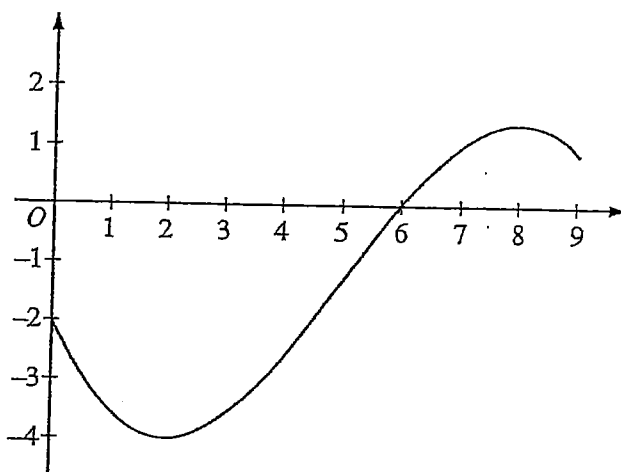
- (A) f is continuous but not differentiable at $x = 2$.
 (B) f is differentiable at $x = 2$.
 (C) f is not continuous at $x = 2$.
 (D) $\lim_{x \rightarrow 2} f(x) \neq 0$
 (E) $x = 2$ is a vertical asymptote of the graph of f .

12. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ is equal to which of the following?

- (A) $2 \int_1^{16} e^u du$ (B) $2 \int_1^4 e^u du$ (C) $2 \int_1^2 e^u du$ (D) $\frac{1}{2} \int_1^2 e^u du$ (E) $\int_1^4 e^u du$

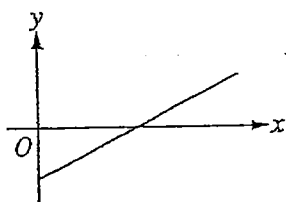
13. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x-1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x) dx$?
- (A) 2 (B) 6 (C) 8 (D) 10 (E) 12

14. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is
- (A) $\frac{7}{\sqrt{5}}$ (B) $\frac{14}{\sqrt{5}}$ (C) $\frac{18}{\sqrt{5}}$ (D) $\frac{15}{\sqrt{21}}$ (E) $\frac{30}{\sqrt{21}}$



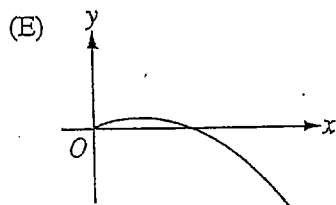
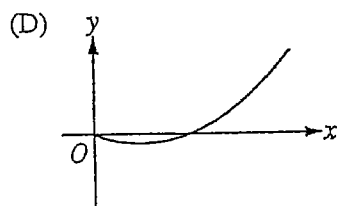
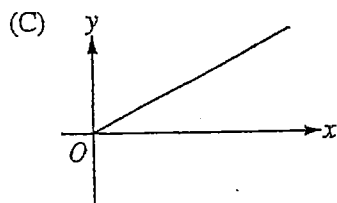
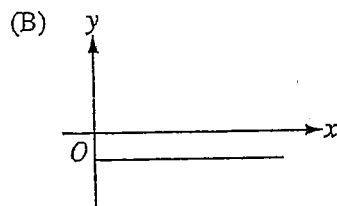
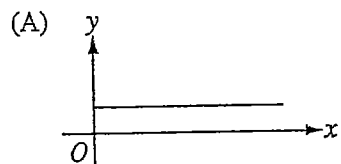
Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t) dt$, which of the following is true?
- (A) $h(6) < h'(6) < h''(6)$
 (B) $h(6) < h''(6) < h'(6)$
 (C) $h'(6) < h(6) < h''(6)$
 (D) $h''(6) < h(6) < h'(6)$
 (E) $h''(6) < h'(6) < h(6)$
16. A particle moves along the x -axis with its position at time t given by $x(t) = (t - a)(t - b)$, where a and b are constants and $a \neq b$. For which of the following values of t is the particle at rest?
- (A) $t = ab$
 (B) $t = \frac{a+b}{2}$
 (C) $t = a + b$
 (D) $t = 2(a + b)$
 (E) $t = a$ and $t = b$



Graph of f

17. The figure above shows the graph of f . If $f(x) = \int_2^x g(t) dt$, which of the following could be the graph of $y = g(x)$?



18. $\lim_{h \rightarrow 0} \frac{\ln(4+h) - \ln(4)}{h}$ is

- (A) 0 (B) $\frac{1}{4}$ (C) 1 (D) e (E) nonexistent

19. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?

(A) $(0, 0)$ only

(B) $\left(\frac{1}{2}, \frac{1}{5}\right)$ only

(C) $(0, 0)$ and $(-4, 2)$

(D) $(0, 0)$ and $\left(4, \frac{2}{3}\right)$

(E) There are no such points.

20. Let $f(x) = (2x + 1)^3$ and let g be the inverse function of f . Given that $f(0) = 1$, what is the value of $g'(1)$?

(A) $\frac{2}{27}$

(B) $\frac{1}{54}$

(C) $\frac{1}{27}$

(D) $\frac{1}{6}$

(E) 6

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \frac{\sin(5x)}{x}$

(B) $y = 5x$

(C) $y = \frac{1}{x-5}$

(D) $y = \frac{5x}{1-x}$

(E) $y = \frac{20x^2 - x}{1 + 4x^2}$

22. Let f be the function defined by $f(x) = \frac{\ln x}{x}$. What is the absolute maximum value of f ?

(A) 1

(B) $\frac{1}{e}$

(C) 0

(D) $-e$

(E) f does not have an absolute maximum value.

23. If $P(t)$ is the size of a population at time t , which of the following differential equations describes linear growth in the size of the population?

(A) $\frac{dP}{dt} = 200$

(B) $\frac{dP}{dt} = 200t$

(C) $\frac{dP}{dt} = 100t^2$

(D) $\frac{dP}{dt} = 200P$

(E) $\frac{dP}{dt} = 100P^2$

24. Let g be the function given by $g(x) = x^2 e^{kx}$, where k is a constant. For what value of k does g have a critical point at $x = \frac{2}{3}$?

- (A) -3 (B) $-\frac{3}{2}$ (C) $-\frac{1}{3}$ (D) 0 (E) There is no such k .

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 2 \sin x$ with the initial condition $y(\pi) = 1$?

(A) $y = 2 \cos x + 3$

(B) $y = 2 \cos x - 1$

(C) $y = -2 \cos x + 3$

(D) $y = -2 \cos x + 1$

(E) $y = -2 \cos x - 1$

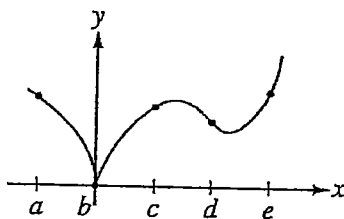
26. Let g be a function with first derivative given by $g'(x) = \int_0^x e^{-t^2} dt$. Which of the following must be true on the interval $0 < x < 2$?

- (A) g is increasing, and the graph of g is concave up.
(B) g is increasing, and the graph of g is concave down.
(C) g is decreasing, and the graph of g is concave up.
(D) g is decreasing, and the graph of g is concave down.
(E) g is decreasing, and the graph of g has a point of inflection on $0 < x < 2$.

27. If $(x + 2y) \cdot \frac{dy}{dx} = 2x - y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$?
- (A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$ (E) Undefined

28. For $t \geq 0$, the position of a particle moving along the x -axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0?
- (A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

Part 2: A graphing calculator is required for these problems.



Graph of f

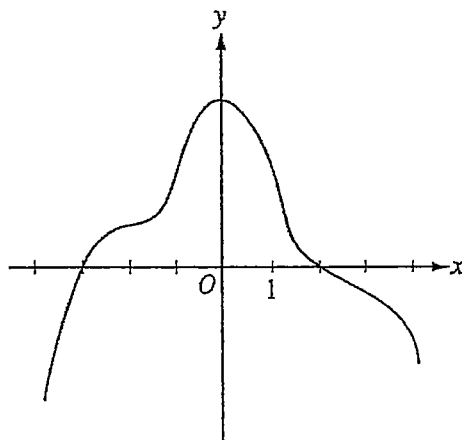
76. The graph of the function f is shown in the figure above. For which of the following values of x is $f'(x)$ positive and increasing?
- (A) a (B) b (C) c (D) d (E) e
77. Let f be a function that is continuous on the closed interval $[2, 4]$ with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem?
- (A) $f(x) = 13$ has at least one solution in the open interval $(2, 4)$.
- (B) $f(3) = 15$
- (C) f attains a maximum on the open interval $(2, 4)$.
- (D) $f'(x) = 5$ has at least one solution in the open interval $(2, 4)$.
- (E) $f'(x) > 0$ for all x in the open interval $(2, 4)$.

78. The graph of $y = e^{\tan x} - 2$ crosses the x -axis at one point in the interval $[0, 1]$. What is the slope of the graph at this point?

- (A) 0.606 (B) 2 (C) 2.242 (D) 2.961 (E) 3.747

79. A particle moves along the x -axis. The velocity of the particle at time t is given by $v(t)$, and the acceleration of the particle at time t is given by $a(t)$. Which of the following gives the average velocity of the particle from time $t = 0$ to time $t = 8$?

- (A) $\frac{a(8) - a(0)}{8}$
 (B) $\frac{1}{8} \int_0^8 v(t) dt$
 (C) $\frac{1}{8} \int_0^8 |v(t)| dt$
 (D) $\frac{1}{2} \int_0^8 v(t) dt$
 (E) $\frac{v(0) + v(8)}{2}$



Graph of f'

80. The graph of f' , the derivative of the function f , is shown above. Which of the following statements must be true?

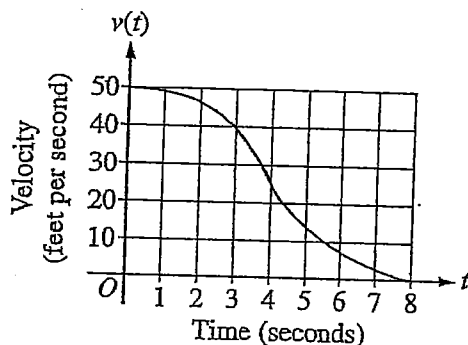
- I. f has a relative minimum at $x = -3$.
 II. The graph of f has a point of inflection at $x = -2$.
 III. The graph of f is concave down for $0 < x < 4$.
- (A) I only (B) II only (C) III only (D) I and II only (E) I and III only

81. Water is pumped into a tank at a rate of $r(t) = 30(1 - e^{-0.16t})$ gallons per minute, where t is the number of minutes since the pump was turned on. If the tank contained 800 gallons of water when the pump was turned on, how much water, to the nearest gallon, is in the tank after 20 minutes?

- (A) 380 gallons
- (B) 420 gallons
- (C) 829 gallons
- (D) 1220 gallons
- (E) 1376 gallons

82. If $f'(x) = \sqrt{x^4 + 1} + x^3 - 3x$, then f has a local maximum at $x =$

- (A) -2.314
- (B) -1.332
- (C) 0.350
- (D) 0.829
- (E) 1.234

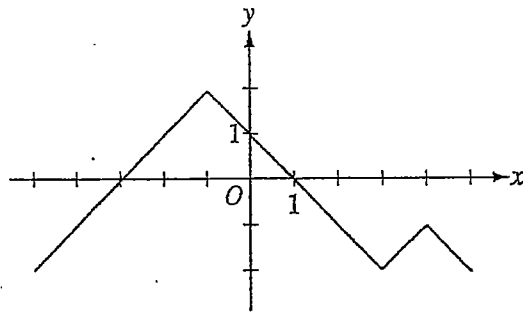


83. The graph above gives the velocity, v , in ft/sec, of a car for $0 \leq t \leq 8$, where t is the time in seconds. Of the following, which is the best estimate of the distance traveled by the car from $t = 0$ until the car comes to a complete stop?

- (A) 21 ft
- (B) 26 ft
- (C) 180 ft
- (D) 210 ft
- (E) 260 ft

84. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{(x^4 - 2x^2 + 1)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

- (A) $(-0.418, 0.418)$ only
- (B) $(-1, 1)$
- (C) $(-1.354, -0.409)$ and $(0.409, 1.354)$
- (D) $(-1.5, -1)$ and $(0, 1)$
- (E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$



Graph of f'

85. The graph of f' , the derivative of f , is shown in the figure above. The function f has a local maximum at $x =$

- (A) -3 (B) -1 (C) 1 (D) 3 (E) 4

86. If $f'(x) > 0$ for all real numbers x and $\int_4^7 f(t) dt = 0$, which of the following could be a table of values for the function f ?

(A)

x	$f(x)$
4	-4
5	-3
7	0

(C)

x	$f(x)$
4	-4
5	6
7	3

(E)

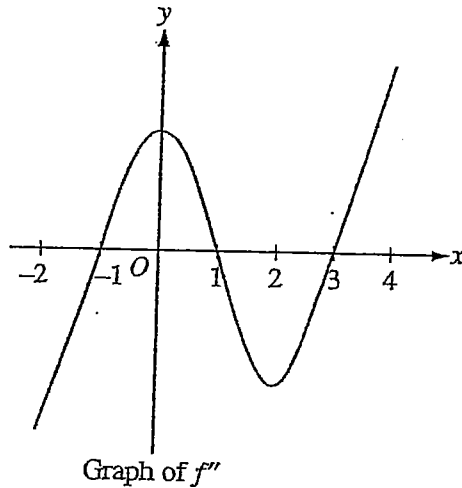
x	$f(x)$
4	0
5	4
7	6

(B)

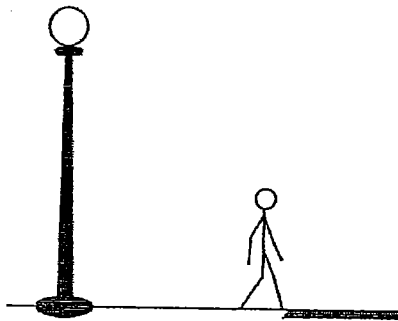
x	$f(x)$
4	-4
5	-2
7	5

(D)

x	$f(x)$
4	0
5	0
7	0



87. The graph of f'' , the second derivative of f , is shown above for $-2 \leq x \leq 4$. What are all intervals on which the graph of the function f is concave down?
- (A) $-1 < x < 1$
 (B) $0 < x < 2$
 (C) $1 < x < 3$ only
 (D) $-2 < x < -1$ only
 (E) $-2 < x < -1$ and $1 < x < 3$



88. A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?
- (A) 1.5 ft/sec (B) 2.667 ft/sec (C) 3.75 ft/sec (D) 6 ft/sec (E) 10 ft/sec
89. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \frac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?
- (A) 0.713 (B) 1.134 (C) 6.134 (D) 6.710 (E) 11.710

90. Let f be a function such that $\int_6^{12} f(2x) dx = 10$. Which of the following must be true?

(A) $\int_{12}^{24} f(t) dt = 5$

(B) $\int_{12}^{24} f(t) dt = 20$

(C) $\int_6^{12} f(t) dt = 5$

(D) $\int_6^{12} f(t) dt = 20$

(E) $\int_3^6 f(t) dt = 5$

x	-2	0	3	5	6
$f'(x)$	3	1	4	7	5

91. Let f be a polynomial function with values of $f'(x)$ at selected values of x given in the table above. Which of the following must be true for $-2 < x < 6$?

(A) The graph of f is concave up.

(B) The graph of f has at least two points of inflection.

(C) f is increasing.

(D) f has no critical points.

(E) f has at least two relative extrema.

92. Let R be the region in the first quadrant bounded below by the graph of $y = x^2$ and above by the graph of $y = \sqrt{x}$. R is the base of a solid whose cross sections perpendicular to the x -axis are squares. What is the volume of the solid?

(A) 0.129

(B) 0.300

(C) 0.333

(D) 0.700

(E) 1.271

**Answer Key for AP Calculus AB
Practice Exam, Section I**

Multiple-Choice Questions	
Question #	Key
1	B
2	B
3	A
4	E
5	C
6	D
7	E
8	C
9	E
10	A
11	A
12	C
13	D
14	A
15	A
16	B
17	A
18	B
19	C
20	D
21	E
22	B

23	A
24	A
25	E
26	A
27	A
28	A
76	E
77	A
78	D
79	B
80	E
81	D
82	C
83	D
84	D
85	C
86	B
87	E
88	B
89	E
90	B
91	B
92	A