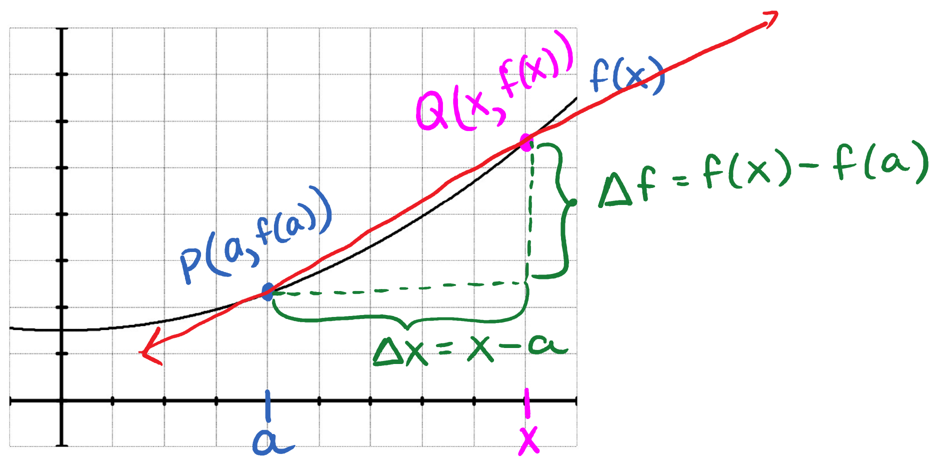
3.1 new

Wednesday, June 21, 2023 1:45 PM

3.1 Definition of a Derivative



Slope of the secant line = $\frac{\Delta f}{\Delta x} = \frac{f(x)-f(a)}{x-a}$

As Q moves towards P the secant line get closer to the slope of the tangent line. Therefore, the slope of the secant line approaches the slope of the tangent line.

The slope of the tangent line at a point is reffered to as the derivative at a point. $\boxed{f'(a)}$ will be the slope of the tangent line.

f prime of a
derivative at a

Equation #1	Equation #2
$f'(a) = \lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{h}$	$f'(a) = \lim\limits_{x \to a} \frac{f(x)-f(a)}{x-a}$

Equation of a tangent line

$$y - y_1 = m(x - x_1) \qquad or \qquad y - f(a) = f'(a)(x-a)$$

m = slope
(x_1, y_1) point

$f'(a)$ = slope
$(a, f(a))$ point

AP Calculus

1. Find $f'(-1)$ if $f(x) = 3x^2 + 2x$ using both equation 1 and equation 2

Equation 1 $f'(a) = \lim\limits_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$$f'(-1) = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$$

$$= \lim_{h \to 0} \frac{3(-1+h)^2 + 2(-1+h) - (3(-1)^2 + 2(-1))}{h}$$

$$= \lim_{h \to 0} \frac{3(1-2h+h^2) - 2 + 2h - (3-2)}{h}$$

$$= \lim_{h \to 0} \frac{3 - 6h + 3h^2 - 2 + 2h - 1}{h}$$

$$= \lim_{h \to 0} \frac{3h^2 - 4h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3h-4)}{\cancel{h}}$$

$$= \lim_{h \to 0} 3h - 4$$

$$= 3(0) - 4$$

$$= -4$$

Equation 2 $f'(a) = \lim\limits_{x \to a} \frac{f(x) - f(a)}{x - a}$

$$f'(-1) = \lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)}$$

$$= \lim_{x \to -1} \frac{3x^2 + 2x - (3(-1)^2 + 2(-1))}{x+1}$$

$$= \lim_{x \to -1} \frac{3x^2 + 2x - (3-2)}{x+1}$$

$$= \lim_{x \to -1} \frac{3x^2 + 2x - 1}{x+1}$$

$3 \times -1 = -3$
$3 + -1 = 2$

$$= \lim_{x \to -1} \frac{3x^2 + 3x - 1x - 1}{x+1}$$

$$= \lim_{x \to -1} \frac{3x(x+1) - 1(x+1)}{x+1}$$

$$= \lim_{x \to -1} \frac{\cancel{(x+1)}(3x-1)}{\cancel{x+1}}$$

$$= \lim_{x \to -1} 3x - 1$$

$$= 3(-1) - 1$$

$$= -4$$

2. Find the equation of the tangent line for $f(x) = \frac{1}{x}$ at x = 3. Use equation 1 to find the derivative.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$y - y_1 = m(x - x_1)$$
or
$$y - f(a) = f'(a)(x - a)$$

Use equation 1 to find the slope of the tangent line

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{3}{(3+h)(3)} - \frac{3+h}{(3+h)(3)}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{3-3-h}{(3+h)(3)}\right]$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}}\left[\frac{-\cancel{h}}{(3+h)(3)}\right]$$

$$= \lim_{h \to 0} \frac{-1}{(3+h)(3)}$$

$$= \frac{-1}{(3+0)(3)}$$

$$= \frac{-1}{9}$$ slope of the tangent line

point

$x = 3$

y-value $f(3) = \frac{1}{3}$

$(3, \frac{1}{3})$

slope $= -\frac{1}{9}$

$$y - \frac{1}{3} = -\frac{1}{9}(x - 3)$$

$$y = -\frac{1}{9}x + \frac{3}{9} + \frac{1}{3}$$

$$y = -\frac{1}{9}x + \frac{1}{3} + \frac{1}{3}$$

$$y = -\frac{1}{9}x + \frac{2}{3}$$

3. Find $f'(0)$ if $f(x) = \sqrt{1+x}$ Use equation 2 $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$$

$$= \lim_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1+0}}{x}$$

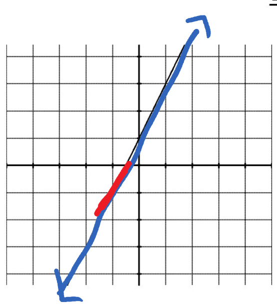
$$= \lim_{x\to 0} \frac{\sqrt{1+x} - 1}{x} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$= \lim_{x\to 0} \frac{1+x+\sqrt{1+x}-\sqrt{1+x}-1}{x(\sqrt{1+x}+1)}$$

$$= \lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{1+x}+1)}$$

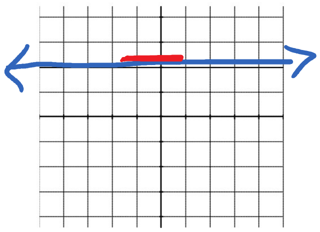
$$= \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}$$

Linear and Constant functions



$\boxed{f(x) = mx + b}$

$f'(a) = m$ for a linear function



$f(x) = b$

$f'(a) = 0$ for a constant function