3.1 New

Tuesday, June 27, 2023 11:21 AM

3.1 Characteristics of Polynomial Functions

Polynomial Function: A function of the form: $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ \ldots\ + a_1 x^1 + a_0$

Where: n is a whole number

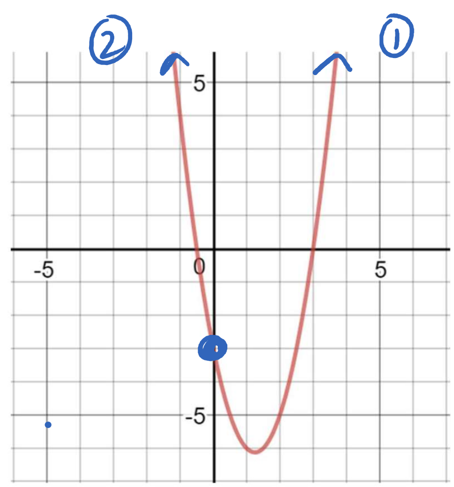
x is a variable

$a_n, a_{n-1}, a_{n-2} \ldots a_0$ are real numbers coefficients

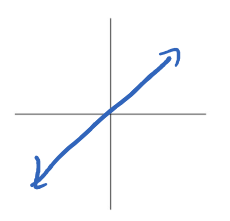
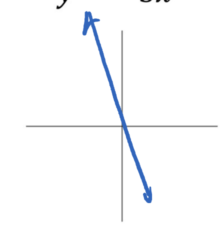
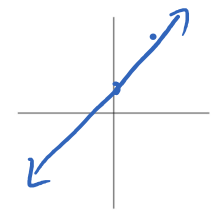
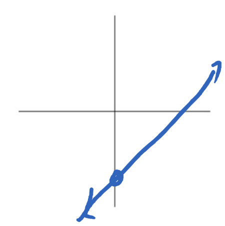
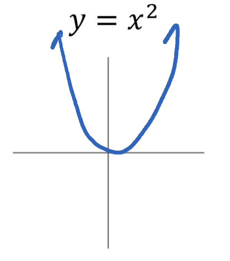
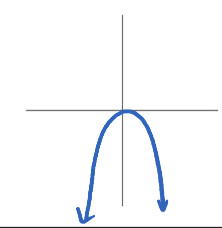
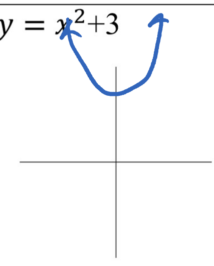
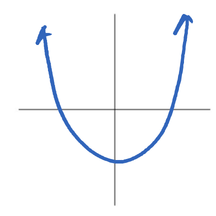
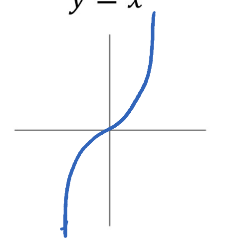
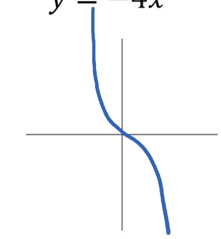
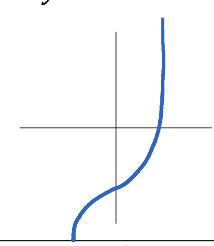
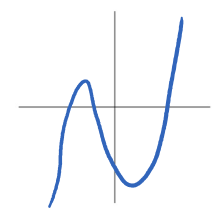
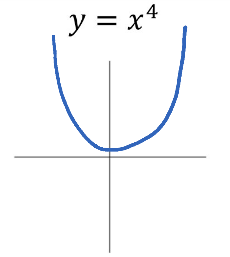
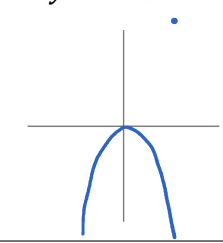
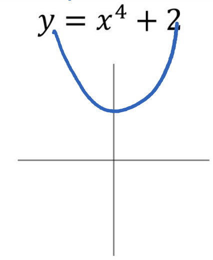
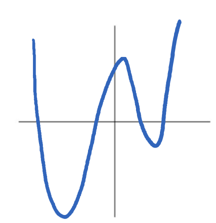
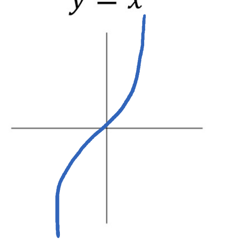
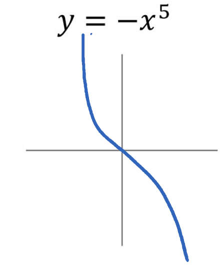
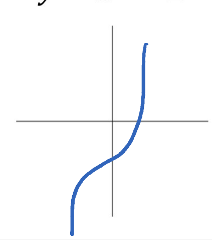
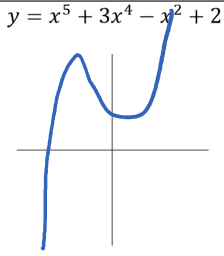
Degree	Highest exponent of the polynomial
Constant term	Term with no variables (y-intercept)
Number of possible x-intercepts	Max # of intercepts = degree
Leading coefficient	coefficient of the highest degree term. (+) or (−)
End behavior	direction and Quadrant that the polynomial is extending into

Example 1: Using the equation and the graph of $f(x) = 2x^{2} - 5x - 3$, find the following:

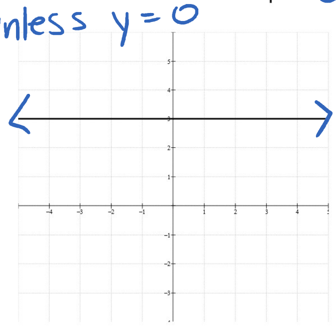
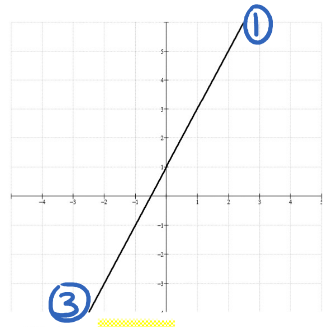
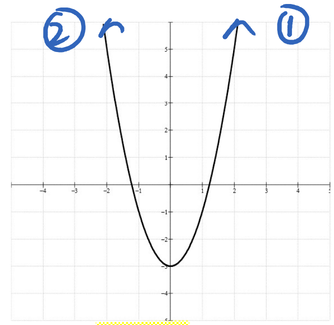
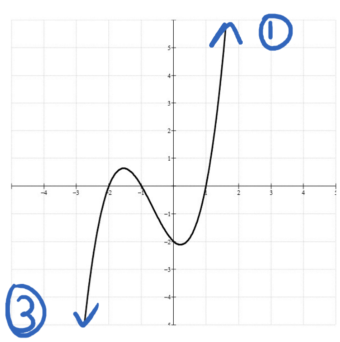
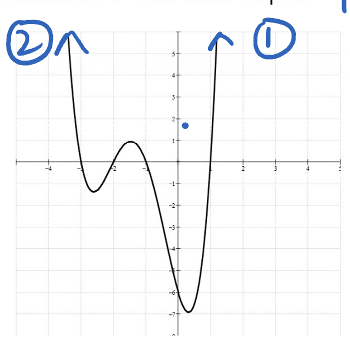
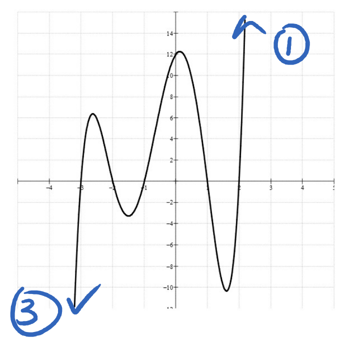
Degree	2
Constant term	−3
Number of possible x-intercepts	2 = degree
Leading coefficient	2 (+) pos
End behavior	up into Quad 1 and Quad 2



Example 2: Use a graphing calculator or the Desmos app to sketch of each function. Look for patterns that will allow you to determine the degree, constant term, number of possible x-intercepts, leading coefficient and end behavior of any function without graphing it.

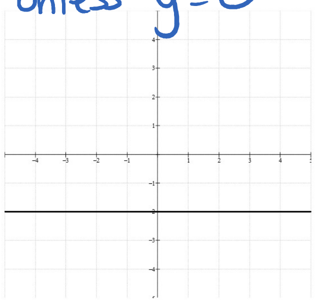
Type				
Linear	$y = x$ 	$y = -3x$ 	$y = x + 1$ 	$y = x - 4$ 
Quadratic	$y = x^2$ 	$y = -2x^2$ 	$y = x^2 + 3$ 	$y = x^2 - x - 2$ 
Cubic	$y = x^3$ 	$y = -4x^3$ 	$y = x^3 - 4$ 	$y = x^3 + 4x^2 + x - 6$ 
Quartic	$y = x^4$ 	$y = -2x^4$ 	$y = x^4 + 2$ 	$y = x^4 + x^3 - 7x^2 + 8$ 
Quintic	$y = x^5$ 	$y = -x^5$ 	$y = x^5 - 1$ 	$y = x^5 + 3x^4 - x^2 + 2$ 

Polynomial Characteristics

Degree 0: **constant** Leading Coefficient: **N/A** Maximum # of x-intercepts: 0 unless y = 0  Ex: $f(x) = 3$ End Behavior: Horizontal line	Degree 1: **linear** Leading Coefficient: **positive** Maximum # of x-intercepts: 1  Ex: $f(x) = 2x + 1$ End Behavior: up into Quad 1 down into Quad 3	Degree 2: **quadratic** Leading Coefficient: **positive** Maximum # of x-intercepts: 2  Ex: $f(x) = 2x^2 - 3$ End Behavior: up into Quad 1 and Quad 2
Degree 3: **cubic** Leading Coefficient: **positive** Maximum # of x-intercepts: 3  Ex: $f(x) = x^3 + 2x^2 - x - 2$ End Behavior: up into Quad 1 down into Quad 3	Degree 4: **quartic** Leading Coefficient: **positive** Maximum # of x-intercepts: 4  Ex: $f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$ End Behavior: up into Quad 1 and Quad 2	Degree 5: **quintic** Leading Coefficient: **positive** Maximum # of x-intercepts: 5  Ex: $f(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$ End Behavior: up into Quad 1 down into Quad 3

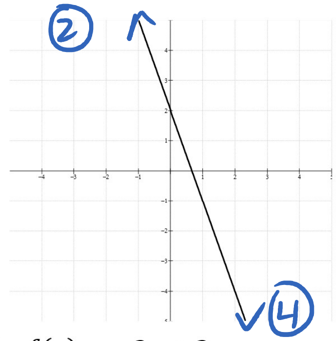
<table>
<tr>
<td>Degree 0: constant

Leading Coefficient: N/A

Maximum # of x-intercepts: 0
unless $y=0$

Ex: $f(x) = -2$

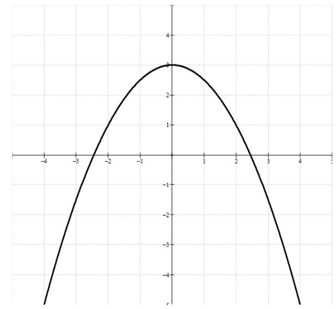
End Behavior:
Horizontal line</td>
<td>Degree 1: linear

Leading Coefficient: negative

Maximum # of x-intercepts: 1

Ex: $f(x) = -3x + 2$

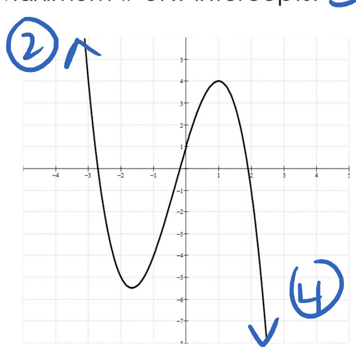
End Behavior:
up into Quad 2
down into Quad 4</td>
<td>Degree 2: quadratic

Leading Coefficient: negative

Maximum # of x-intercepts: 2

Ex: $f(x) = -\frac{1}{2}x^2 + 3$

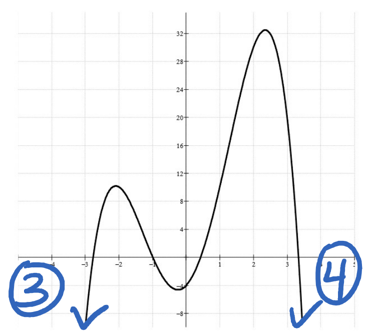
End Behavior:
Down into
Quad 3 and 4</td>
</tr>
<tr>
<td>Degree 3: cubic

Leading Coefficient: negative

Maximum # of x-intercepts: 3

Ex: $f(x) = -x^3 - x^2 + 5x + 1$

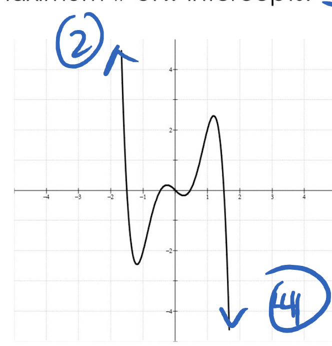
End Behavior:
up into Quad 2
down into Quad 4</td>
<td>Degree 4: quartic

Leading Coefficient: negative

Maximum # of x-intercepts: 4

Ex: $f(x) = -x^4 + 10x^2 + 5x - 4$

End Behavior:
Down into Quad
3 and 4</td>
<td>Degree 5: quintic

Leading Coefficient: negative

Maximum # of x-intercepts: 5

Ex: $f(x) = -2x^5 + 5x^3 - x$

End Behavior:
Up into Quad 2
down into Quad 4</td>
</tr>
</table>

- Even-numbered degrees have the same end behavior
- Odd-numbered degrees have the same end behavior

Example 3: Without graphing, describe the function $y = -x^4 + 10x^2 + 5x - 2$.

Example 3: Without graphing, describe the function $y = -x^4 + 10x^2 + 5x - 2$.

Degree	4
Constant term	−2 (y-intercept)
Number of possible x-intercepts	4 (same as degree)
Leading coefficient	− or neg or −1
End behavior	down into Quad 3, and 4
Direction of opening	down

Example 4: Without graphing, describe the function $y = x^3 + x^2 - 5x + 8$.

Degree	3
Constant term	8
Number of possible x-intercepts	3
Leading coefficient	1 or pos or +
End behavior	up into Quad 1 down into Quad 3
Direction of opening	N/A

Practice: p.114 #1 – 5, [redacted]