3.2 New

Tuesday, June 27, 2023 11:36 AM

AP Calculus

3.2 The Derivative as a Function

We used the equation $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ to calculate the derivative at a point. Now we are going to find the derivative as a function.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Pascals Δ

1
1 2 1
1 3 3 1
1 4 6 4 1

$(a+b)^2 = 1a^2 + 2ab + 1b^2$

1. Find the derivative of $f(x) = x^3 + 2x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^3 + 2(x+h) - (x^3+2x)}{h}$$

$$= \lim_{h\to 0} \frac{x^3 + 3x^2h^1 + 3x^1h^2 + 1h^3 + 2x + 2h - x^3 - 2x}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 + 2h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 + 2)}{\cancel{h}}$$

$$= \lim_{h\to 0} 3x^2 + 3xh + h^2 + 2$$

$$= 3x^2 + 3x(0) + 0^2 + 2 \quad = 3x^2 + 2$$

Notation for derivatives

Function	derivative
$f(x)$	$f'(x)$
y	y'
y	$\frac{dy}{dx}$

Differentiability

A function is differentiable on (a,b) if the domain of $f'(x)$ consists of all values x of the domain of $f(x)$ for which $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ exists.

2. Prove that $f(x)=\frac{1}{x}$ is differentiable. $f(x)=\frac{1}{x}$ for $x\neq 0$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{\frac{h}{1}}$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}\right]$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{x-x-h}{x(x+h)}\right]$$

$$=\lim_{h\to 0}\frac{1}{\cancel{h}}\left[\frac{-\cancel{h}}{x(x+h)}\right]$$

$$=\lim_{h\to 0}\frac{-1}{x(x+h)}$$

$$=\frac{-1}{x(x+0)}=-\frac{1}{x^2}\qquad x\neq 0$$

f(x) is differentiable
same restriction for
f(x) and f'(x)

Power Rule

$$\frac{d}{dx}x^n=nx^{n-1}$$

$\frac{3}{4} - 1$

$\frac{3}{4} - \frac{4}{4}$

3. Find the derivative using the power rule

a) $y = x^5$

$y' = 5x^4$

b) $y = x^{\frac{3}{4}}$

$y = \frac{3}{4}x^{-\frac{1}{4}}$

Sum and Difference Rule

$(f \pm g)' = f' \pm g'$

Constant Multiple Rule

$(cf)' = cf'$

4. Find the derivative of $g(x) = 7x^{-3} + 4x^2 + 5$

$g'(x) = 7(-3)x^{-4} + 4(2)x^1 + 0$

$g'(x) = -21x^{-4} + 8x$

5. Find the **points** where the tangent lines to $f(x) = x^3 - 27x + 4$ are horizontal.

(x, y) Horizontal tangent line $m = 0$

Find where $f'(x) = 0$

$f'(x) = 3x^2 - 27$

$0 = 3x^2 - 27$

$0 = x^2 - 9$

$0 = (x+3)(x-3)$

$x = -3 \quad x = 3$

$f(3) = 3^3 - 27(3) + 4$

$f(3) = 27 - 81 + 4$

$f(3) = -50$

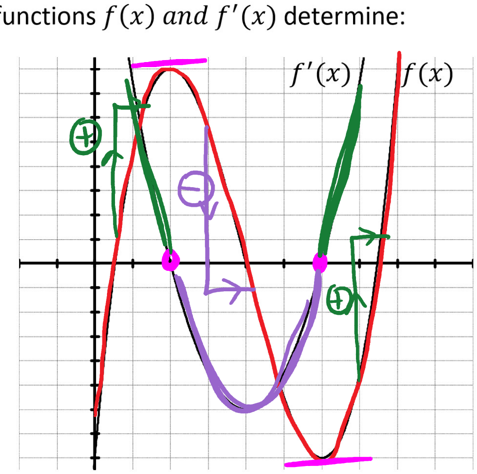
$f(-3) = (-3)^3 - 27(-3) + 4$

$f(-3) = -27 + 81 + 4$

$f(-3) = 58$

$(3, -50) \quad (-3, 58)$

6. Given the functions $f(x)$ and $f'(x)$ determine:



m= 0

a) When is the slope of the tangent line equal to zero?

$x=2$ $\quad$ $x=6$

b) When is the slope of the tangent line positive?

$0<x<2$ $\quad$ $x>6$

c) When is the slope of the tangent line negative?

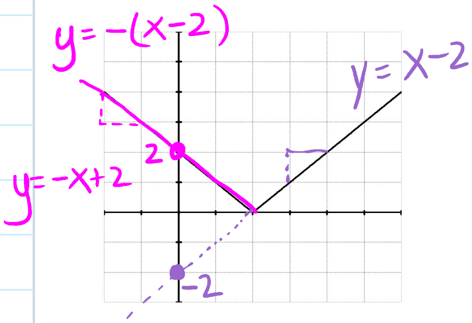
$2<x<6$

<u>Differentiability Implies Continuity</u>

If *f* is differentiable at *x=c,* then *f* is continuous at *x=c*

However continuity does not gaurantee differentiablity

7. Show that $f(x) = |x - 2|$ is continuous but not differentiable at $x = 2$



Continuity:

1. $f(2) = |2-2| = 0$

2. $\lim_{x\to 2^-} f(x) = 0 \quad \lim_{x\to 2^+} f(x) = 0$

$\lim_{x\to 2} f(x) = 0$

3. $f(2) = \lim_{x\to 2} f(x)$

$0 = 0$

Find $f'(x)$ using equation 2 $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

Limit from left

$\lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$

$\lim_{x\to 2^-} \frac{|x-2|-0}{x-2}$

$\lim_{x\to 2^-} \frac{-\cancel{(x-2)}}{\cancel{x-2}}$

$\lim_{x\to 2^-} -1$

$= -1$

Limit from right

$\lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2}$

$\lim_{x\to 2^+} \frac{|x-2|-0}{x-2}$

$\lim_{x\to 2^+} \frac{\cancel{x-2}}{\cancel{x-2}}$

$\lim_{x\to 2^+} 1$

$= 1$

left $\neq$ right

$\therefore$ limit does not exist

$f(x)$ is NOT differentiable at $x=2$