3.2 Remainder Theorem

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3.2 The Remainder Theorem

Recall long division with numbers:

Quotient → 66

divisor → 8) 532 ← Dividend

48

52

48

4 ← Remainder

A. Long Division with Polynomials

Example 1: Divide $2x^3 - 4x^2 + 3x - 6$ by $x + 2$

$$\begin{array}{r} 2x^2 - 8x + 19 \leftarrow \text{Quotient} \\ x + 2 \,\overline{)\, 2x^3 - 4x^2 + 3x - 6} \\ 2x^3 + 4x^2 \\ \hline -8x^2 + 3x \\ -8x^2 - 16x \\ \hline 19x - 6 \\ 19x + 38 \\ \hline -44 \text{ Remainder} \end{array}$$

$2x^3 - 2x^3$

$-4x^2 - 4x^2$

$-8x^2 - (-8x^2)$

$3x - (-16x)$

$19x - 19x$

$-6 - 38$

Division Statement: When $P(x)$ ← Dividend is divided by $x - a$ ← Divisor

$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$ or $P(x) = Q(x)(x-a) + R$

$$\frac{2x^3 - 4x^2 + 3x - 6}{x + 2} = 2x^2 - 8x + 19 + \frac{(-44)}{x+2}$$

$2x^3 - 4x^2 + 3x - 6 = (2x^2 - 8x + 19)(x + 2) + (-44)$

Write the terms in order
Hold all positions

Example 2: Divide $\frac{x^3-6-6x}{x-2}$

$$
\begin{array}{r|l}
 & x^2+2x-2 \\
x-2 & x^3+0x^2-6x-6 \\
 & x^3-2x^2 \\
 & \quad 2x^2-6x \\
 & \quad 2x^2-4x \\
 & \qquad -2x-6 \\
 & \qquad -2x+4 \\
 & \qquad\quad -10
\end{array}
$$

$0x^2-(-2x^2)$

$-6x-(-4x)$

$-6-4$

B. Synthetic Division with Polynomials $\boxed{\text{Trick}}$

Synthetic division is an alternative method of polynomial division.

Example 3: $(x^3+2x^2-7x-2)\div(x+1)$

$$
\begin{array}{r|rrrr}
-1 & 1 & 2 & -7 & -2 \\
 & & (-1)(1) & (-1)(3) & (-1)(-4) \\
 & & -1 & -3 & 4 \\
\hline
 & 1 & 3 & -4 & -6
\end{array}
$$

$2-(-1)$ $\quad -7-(-3)$ $\quad -2-4$

Quotient $= 1x^2+3x-4$

Remainder $= -6$

Example 4: $(x^3 - 5x^2 + 10x - 15) \div (x - 3)$

$R = -3$

$$\begin{array}{r|rrrr} -3 & 1 & -5 & 10 & -15 \\ & & -3(1) & -3(-2) & -3(4) \\ & & -3 & 6 & -12 \\ \hline & 1 & -2 & 4 & \boxed{-3} \end{array}$$

$-5-(-3)$ $10-6$ $-15-(-12)$

$Q(x) = 1x^2 - 2x + 4$ $R = -3$

C. The Remainder Theorem

The Remainder Theorem states that when a polynomial $P(x)$ is divided by $(x - a)$, the remainder is $P(a)$.

$R = P(a)$

Remainder $R = P(a)$ when dividing $P(x)$ by $(x - a)$.

Example 5: Find the remainder when $P(x) = x^3 - 8x^2 + x + 37$ is divided by $(x - 2)$.

$R = P(a)$

$R = P(2)$

$R = (2)^3 - 8(2)^2 + (2) + 37$

$R = 8 - 32 + 2 + 37$

$R = 15$

$x - 2 = x - a$

$-2 = -a$

$2 = a$

Example 6: When $P(x) = x^3 - \boldsymbol{k}x^2 + 17x + 6$ is divided by $(x - 3)$, the remainder is 12. Find $\boldsymbol{k}$.

divisor

$R = 12$

$x - 3 = x - a$
$-3 = -a$
$3 = a$

$R = P(a)$

$12 = P(3)$

$12 = 3^3 - k(3)^2 + 17(3) + 6$

$12 = 27 - 9k + 51 + 6$

$12 = -9k + 84$

$-84 \qquad -84$

$-72 = -9k$

$8 = k$

Practice: p.124 #3cd, 4de, 6ac, 7ab, 8cd, 10,

Mrs. Shaw PC 12