

### 3.3 New

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1:45 PM

AP Calculus

#### 3.3 Product and Quotient Rule

Product Rule: If  $f(x)$  and  $g(x)$  are differentiable functions then

$$(fg)'(x) = f(x)g'(x) + g(x)f'(x)$$

$$(1st)(2nd)' + (2nd)(1st)'$$

1. Find the derivative of  $h(x) = (3x - 2x^2)(5 + 4x)$

$$h'(x) = (3x - 2x^2)(4) + (5 + 4x)(3 - 4x)$$

$$h'(x) = 12x - 8x^2 + 15 - 20x + 12x - 16x^2$$

$$h'(x) = -24x^2 + 4x + 15$$

2. Find the derivative of  $h(x) = \sqrt{x}(1 - x^4) = x^{\frac{1}{2}}(1 - x^4)$

$$h'(x) = x^{\frac{1}{2}} \cdot (-4x^3) + (1 - x^4) \cdot \frac{1}{2}x^{-\frac{1}{2}}$$

$$h'(x) = -4x^{\frac{7}{2}} + \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{\frac{7}{2}}$$

$$h'(x) = -\frac{9}{2}x^{\frac{7}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$$

$$h'(x) = \frac{x^{-\frac{1}{2}}}{2}(-9x^4 + 1) = \frac{-9x^4 + 1}{2x^{\frac{1}{2}}}$$

$$\frac{1}{2} + 3 = \frac{7}{2}$$

$$4 + (-\frac{1}{2}) = \frac{8}{2} + (-\frac{1}{2}) = \frac{7}{2}$$

$$-4 + (-\frac{1}{2}) = -\frac{8}{2} + (-\frac{1}{2}) = -\frac{9}{2}$$

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3. Find  $\frac{dy}{dx}\bigg|_{x=1}$

$$y = (x^{-1} - x^2)(x^3 + 1)$$
$$y = \left(\frac{1}{x} - x^2\right)(x^3 + 1)$$

① Find  $\frac{dy}{dx}$

② Evaluate the derivative at  $x=1$

$$\frac{dy}{dx} = (x^{-1} - x^2)(3x^2) + (x^3 + 1)(-1x^{-2} - 2x')$$

Sub  $x=1$  into  $\frac{dy}{dx}$

$$\frac{dy}{dx}\bigg|_{x=1} = (1^{-1} - 1^2)(3(1)^2) + (1^3 + 1)(-1(1)^{-2} - 2(1))$$

$$= (1 - 1)(3) + (2)(-1 - 2)$$

$$= (0)(3) + 2(-3)$$

$$= -6$$

Slope of the tangent line when  $x=1$  is  $-6$   $m=-6$

Using a Graphing Calculator

nDeriv (function,  $x$ , value)

$\frac{d}{dx}$  function  $x = \text{value}$

Quotient Rule: If  $f(x)$  and  $g(x)$  are differentiable functions then

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

4. Find the derivative of  $y = \frac{5x-1}{x^2+1}$

$$\frac{dy}{dx} = \frac{(x^2+1)(5) - (5x-1)(2x)}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{5x^2+5-10x^2+2x}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{-5x^2+2x+5}{(x^2+1)^2}$$

5. Find the derivative  $h(x) = \frac{x+4}{x^2+x+1}$

$$h'(x) = \frac{(x^2+x+1)(1) - (x+4)(2x+1)}{(x^2+x+1)^2}$$

$$h'(x) = \frac{x^2+x+1-2x^2-x-8x-4}{(x^2+x+1)^2}$$

$$h'(x) = \frac{-x^2-8x-3}{(x^2+x+1)^2}$$

6. Find the derivative  $y = \left(\frac{x^2-4}{x-1}\right)\left(\frac{x^2-1}{(x+2)^2}\right)$

$$y = \frac{(x-2)\cancel{(x+2)}}{\cancel{x-1}} \cdot \frac{\cancel{(x-1)}(x+1)}{(x+2)^2}$$

$$y = \frac{(x-2)(x+1)}{x+2}$$

$$y = \frac{x^2 - x - 2}{x+2}$$

$$\frac{dy}{dx} = \frac{(x+2)(2x-1) - (x^2-x-2)(1)}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{2x^2 - x + 4x - 2 - x^2 + x + 2}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{x^2 + 4x}{(x+2)^2}$$

$$\frac{x(x+4)}{(x+2)^2}$$