3.3 Factor Theorem

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$$\frac{x^3+2x^2-5x-6}{x+1}$$

$x-a=x+1$
$-a=1$
$a=-1$

Synthetic division

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -5 & -6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

Remainder theorem $R=P(a)$

$P(-1)=(-1)^3+2(-1)^2-5(-1)-6$
$P(-1)=-1+2+5-6$
$P(-1)=0$

Factor Theorem:

$x-a$ is a factor of a polynomial if
$P(a)=0$
Remainder $=0$

1. Determine if $x-5$ is a factor of $P(x)=x^3-2x^2-33x+90$

$x-5=x-a$
$-5=-a$
$5=a$

$P(5)=5^3-2(5)^2-33(5)+90$
$P(5)=125-50-165+90$
$P(5)=0$
$(x-5)$ is a factor

2. Determine if $x+3$ is a factor of $P(x)=x^3-3x^2-x+3$

$a=-3$

$P(-3)=(-3)^3-3(-3)^2-(-3)+3$
$P(-3)=-27-27+3+3$
$P(-3)=-48$
$(x+3)$ is NOT a factor

Integral Zero Theorem: (Possible factors of a polynomial)

If $(x-a)$ is a factor of a polynomial, then "a" is a factor of the constant term.

3. List all the possible values for 'a' such that $(x-a)$ may be a factor of the polynomial.
$P(x) = x^3 + 3x^2 - 2x + 12$

Factors of 12 $\{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12\}$

4. Factor the polynomial $P(x) = x^3 - 6x^2 + 7x + 6$

$\{\pm1, \pm2, \pm3, \pm6\}$

$P(3) = 3^3 - 6(3)^2 + 7(3) + 6$
$P(3) = 27 - 54 + 21 + 6$
$P(3) = 0$

$(x-3)$ is a factor

$$\begin{array}{r|rrrr} -3 & 1 & -6 & 7 & 6 \\ & & -3 & 9 & 6 \\ \hline & 1 & -3 & -2 & 0 \end{array}$$

$P(x) = (x-3)\underbrace{(x^2 - 3x - 2)}_{\text{does not factor}}$

$\_\,x\,\_ = -2$
$\_\,+\,\_ = -3$

$\cancel{-1 \times 2}$
$\cancel{1 \times 2}$

1. List all the possible values for 'a'

2. Use the remainder theorem to find a value such that P(a)=0

3. Use synthetic or long division to find the quotient when P(x) is divided by (x-a). The remainder should be zero.

4. Factor again if possible. Try decomposition if the factor is degree 2. Use these steps again if the factor is degree 3 or higher.

5. Factor the polynomial $P(x) = 2x^3 - 15x^2 + 27x - 10$

$\{\pm 1, \pm 2, \pm 5, \pm 10\}$

$P(2) = 2(2)^3 - 15(2)^2 + 27(2) - 10$

$P(2) = 16 - 60 + 54 - 10$

$P(2) = 0$

$(x-2)$ is a factor

$$\begin{array}{r|rrrr} -2 & 2 & -15 & 27 & -10 \\ & & -4 & 22 & -10 \\ \hline & 2 & -11 & 5 & 0 \end{array}$$

$-15-(-4)$

$P(x) = (x-2)(2x^2 - 11x + 5)$

Decomposition

$-1 \times -10 = 10$

$\_ + \_ = -11$

$P(x) = (x-2)[2x^2 - 1x - 10x + 5]$

$P(x) = (x-2)[x(2x-1) - 5(2x-1)]$

$P(x) = (x-2)(2x-1)(x-5)$

6. Factor the polynomial $P(x) = x^4 - 3x^3 - 7x^2 + 15x + 18$

$\{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18\}$

$P(-1) = 1 + 3 - 7 - 15 + 18$

$P(-1) = 0$. $(x+1)$ is a factor

$$\begin{array}{c|ccccc} 1 & 1 & -3 & -7 & 15 & 18 \\ & & 1 & -4 & -3 & 18 \\ \hline & 1 & -4 & -3 & 18 & 0 \end{array}$$

$-7-(-4)$

$15-(-3)$

$P(x) = (x+1)\underbrace{(x^3 - 4x^2 - 3x + 18)}_{f(x)}$

$f(-2) = (-2)^3 - 4(-2)^2 - 3(-2) + 18$

$f(-2) = -8 - 16 + 6 + 18$

$f(-2) = 0$ $(x+2)$ is a factor

$$\begin{array}{c|cccc} 2 & 1 & -4 & -3 & 18 \\ & & 2 & -12 & 18 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$-3-(-12)$

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$P(x) = (x+1)(x+2)(x^2 - 6x + 9)$

$P(x) = (x+1)(x+2)(x-3)(x-3)$ or

$P(x) = (x+1)(x+2)(x-3)^2$

$$P(x) = (x+1)(x+2)(x-3)^2$$