3.4 New

Thursday, September 14, 2023 2:19 PM

AP Calculus

3.4 Rates of Change

Average Rate of Change $\frac{\Delta y}{\Delta x} = \frac{f(x_1)-f(x_0)}{x_1-x_0}$

Instantaneous rate of Change $f'(x_0) = \lim_{x_1 \to x_0} \frac{f(x_1)-f(x_0)}{x_1-x_0}$

1. Find the rate of change of the volume of a cube with respect to the length of its side *x* when *x=4*

Volume of a cube $V = x^3$

$\frac{dV}{dx} = 3x^2$ $\qquad \left.\frac{dV}{dx}\right|_{x=4} = 3(4)^2 = 48$

$s(t)$ = Position function, gives the position on the axis with respect to time.

$s > 0$ right $\qquad s < 0$ left

$Rate = \frac{distance}{time}$

$Average\ Velocity = \frac{Change\ in\ position}{Change\ in\ time}$

$= \frac{\Delta s}{\Delta t}$

$= \frac{s(t_1) - s(t_0)}{t_1 - t_0}$

2. A billiard ball is dropped from a height of 100 feet. $s(t) = -16t^2 + 100$ where time is in seconds.

a) Find the average velocity on the time interval $[1, 2]$

to t_1

$$\text{Av Vel} = \frac{-16(2)^2+100-(-16(1)^2+100)}{2-1} = \frac{36-84}{1}$$

$$= -48 \text{ ft/sec}$$

b) Find the average velocity on the time interval $[1, 1.5]$

to t_2

$$\text{Av Vel} = \frac{-16(1.5)^2+100-(-16(1)^2+100)}{1.5-1} = \frac{64-84}{0.5}$$

$$= -40 \text{ ft/sec}$$

c) Find the average velocity on the time interval $[1, 1.1]$

to t_1

$$\text{Av Vel} = \frac{-16(1.1)^2+100-(-16(1)^2+100)}{1.1-1} = \frac{80.64-84}{.1}$$

$$= \frac{-3.36}{.1} = -33.6 \text{ ft/sec}$$

<u>Instantaneous Velocity</u>

Instantaneous velocity is like reading the speedometer on a car at any given instant of time

$$v(t) = \lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t}$$

$v(t) = s'(t)$

Velocity = derivative of position

3. A billiard ball is dropped from a height of 100 feet. $s(t) = -16t^2 + 100$ where time is in seconds. Find the instantaneous velocity at 1 second.

$v(t) = s'(t)$

$s'(t) = -32t$

$s'(1) = -32(1)$

$= -32$ ft/sec

Speed= |velocity| speed is positive

Position of a free falling object

$$s(t) = S_0 + V_0 t - \frac{1}{2} g t^2$$

g = acceleration due to gravity

V_0 = Initial velocity

S_0 = Initial Position

$g = 9.8 \text{ m/sec}^2$

$g = 32 \text{ ft/sec}^2$

#4 At time t=0 a diver jumps from a diving board that is 40ft above the water where time is in seconds.

$$s(t) = -16t^2 + 16t + 40$$

What is the diver's maximum height?

Max occurs when velocity = 0

$s'(t)$ = velocity

$s'(t) = -32t + 16$

$0 = -32t + 16$

$32t = 16$

$t = \frac{1}{2}$

$s(\frac{1}{2}) = -16(\frac{1}{2})^2 + 16(\frac{1}{2}) + 40$

$s(\frac{1}{2}) = -4 + 8 + 40$

$s(\frac{1}{2}) = 44$ ft