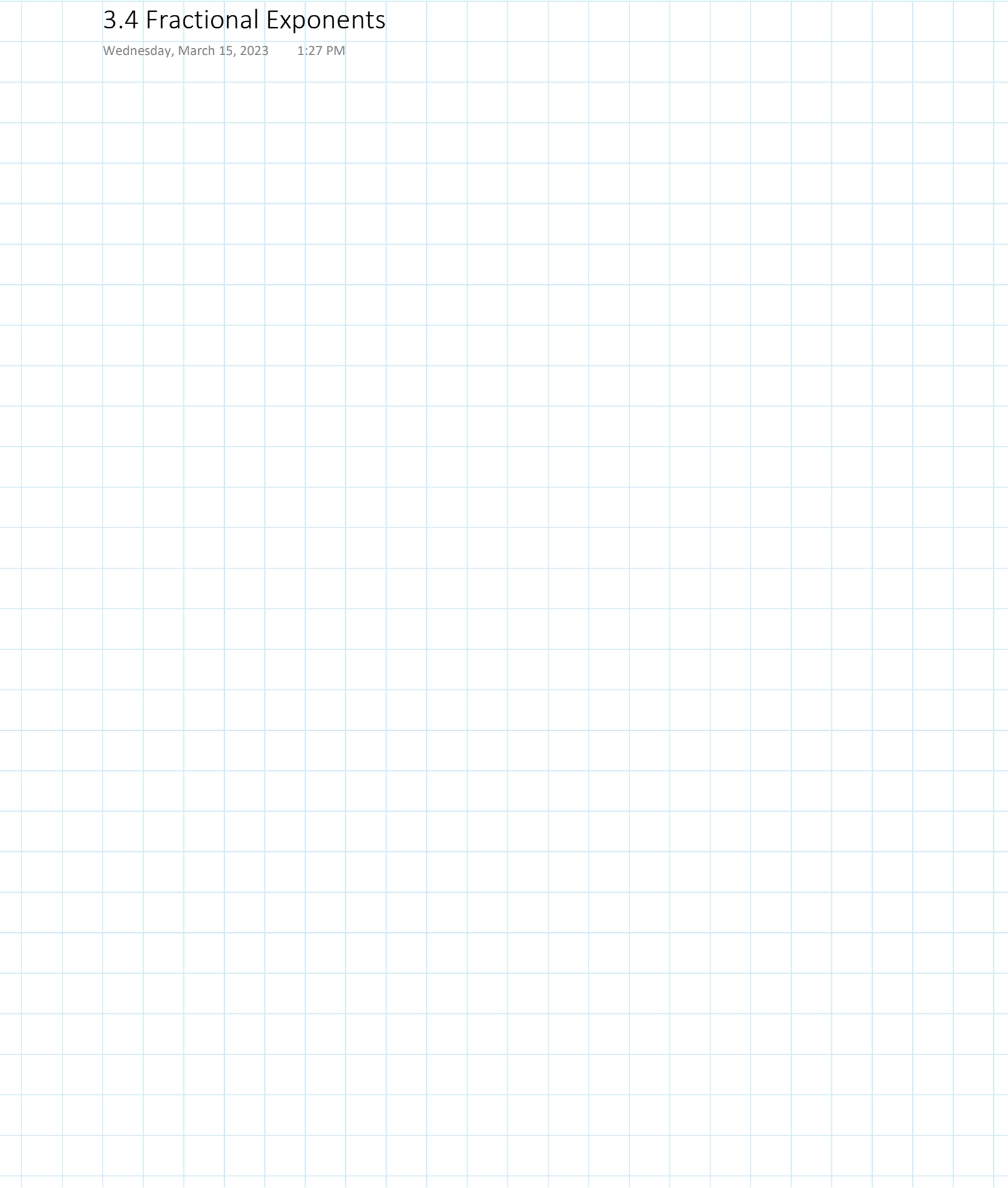


3.4 Fractional Exponents

Wednesday, March 15, 2023 1:27 PM



3.4 Fractional Exponents and Radicals

Radical Form:

Exponential Form:

Use a calculator to complete the following tables.

$$x^{\frac{1}{2}} = \sqrt{x}$$

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} = 1$
4	$4^{\frac{1}{2}} = 2$
9	$9^{\frac{1}{2}} = 3$
16	4
25	5
36	6

x	$x^{\frac{1}{3}}$
1	$1^{\frac{1}{3}} = 1$
8	$8^{\frac{1}{3}} = 2$
27	$27^{\frac{1}{3}} = 3$
64	4
125	5
216	6

Notice a pattern :

\sqrt{x} the square root in exponential form is: $x^{\frac{1}{2}}$

$\sqrt[3]{x}$ the cube root in exponential form is: $x^{\frac{1}{3}}$

In general, $x^{\frac{1}{n}}$ as a radical becomes $\sqrt[n]{x}$.

And vice versa, $\sqrt[n]{x}$ equals $x^{\frac{1}{n}}$.

Radical \uparrow

\uparrow Exponent

Example 1: Write as a radical and then evaluate.

a) $1000^{\frac{1}{3}}$

$$\sqrt[3]{1000} = 10$$

b) $0.25^{\frac{1}{2}}$

$$\begin{aligned} \sqrt{0.25} &= \sqrt{\frac{25}{100}} \\ &= \frac{\sqrt{25}}{\sqrt{100}} \\ &= \frac{5 \div 5}{10 \div 5} \\ &= \frac{1}{2} \end{aligned}$$

c) $(-64)^{\frac{1}{3}}$

$$\sqrt[3]{-64} = -4$$

d) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

$$\begin{aligned} \sqrt[4]{\frac{16}{81}} &= \frac{\sqrt[4]{16}}{\sqrt[4]{81}} \\ &= \frac{2}{3} \end{aligned}$$

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Powers with Rational Exponents

When m and n are natural numbers, and x is a rational number:

$$x^{\frac{m}{n}} = (\sqrt[n]{x})^m \quad \text{or} \quad x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

Example 2: Write $26^{\frac{2}{5}}$ in radical form in two different ways.

i) $(\sqrt[5]{26})^2$ or ii) $\sqrt[5]{26^2}$

Example 3: Write in exponential form.

a) $\sqrt[3]{3^5} = 3^{\frac{5}{3}}$ b) $(\sqrt[3]{25})^2 = 25^{\frac{2}{3}}$

Example 4: Write in radical form and then evaluate.

a) $8^{\frac{2}{3}}$
 $= (\sqrt[3]{8})^2$
 $= (2)^2$
 $= 4$

b) $81^{\frac{3}{4}}$
 $= (\sqrt[4]{81})^3$
 $= (3)^3$
 $= 27$

c) $(-32)^{0.4} = (-32)^{\frac{2}{5}}$
 $\frac{0.4 \times 10}{1 \times 10} = \frac{4}{10} = \frac{2}{5}$
 $(\sqrt[5]{-32})^2 = (-2)^2 = 4$

d) $0.04^{\frac{3}{2}}$
 $\frac{0.04 \times 100}{1 \times 100} = \frac{4}{100}$
 $(\sqrt{\frac{4}{100}})^3 = (\frac{\sqrt{4}}{\sqrt{100}})^3 = (\frac{2}{10})^3 = \frac{2^3}{10^3} = \frac{8}{1000}$
 $\frac{8 \div 8}{1000 \div 8} = \frac{1}{125}$

Practice: p. 227 # 3-8, 10-12, 16

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$$\left(\sqrt[2]{\frac{4 \div 4}{100 \div 4}} \right)$$

$$\frac{8 \div 8}{1000 \div 8}$$
$$\frac{1}{125}$$

$$\left(\sqrt[2]{\frac{1}{25}} \right)^3$$
$$\frac{(\sqrt{1})^3}{(\sqrt{25})^3}$$

$$\frac{1^3}{5^3}$$
$$\frac{1}{125}$$