3.4 Part 1 New

Tuesday, June 27, 2023 11:22 AM

3.4 Equations and Graphs of Polynomial Functions: Part 1

Find the zeros of the function

$f(x) = \frac{1}{2}(x-1)(x+2)(x-3)$

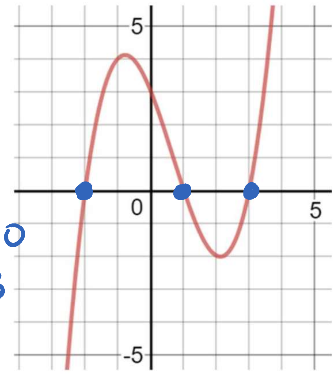
$2(0) = \cancel{2}\frac{1}{2}(x-1)(x+2)(x-3)$

$0 = (x-1)(x+2)(x-3)$

$x - 1 = 0$ $\quad x + 2 = 0$ $\quad x - 3 = 0$

$x = 1$ $\quad x = -2$ $\quad x = 3$

Find the x-intercepts of the function



(−2,0)
(1,0)
(3,0)

The zeroes of a polynomial function are the x-intercepts of the graph of the function.

They are also known as roots

Multiplicity of a zero/root : how many times a particular number is a zero for a given polynomial.

$f(x) = (x-1)^2(x+4)$

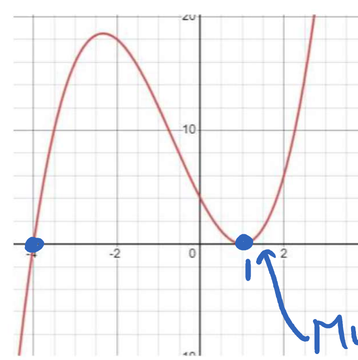
$0 = (x-1)^2(x+4)$

$x - 1 = 0$ $\quad x - 1 = 0$ $\quad x + 4 = 0$

$x = 1$ $\quad x = 1$ $\quad x = -4$

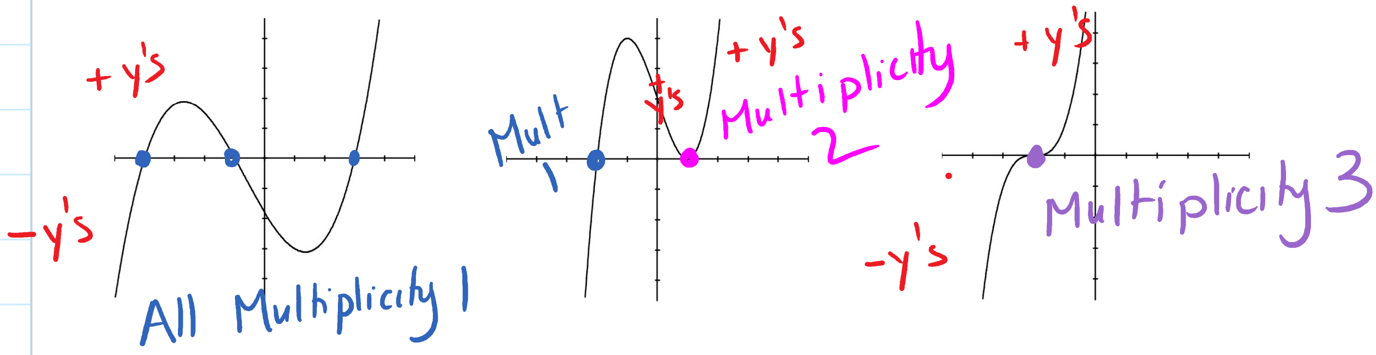
same zero

$x = 1$ Multiplicity 2



Multiplicity 2
zero sits on x-axis

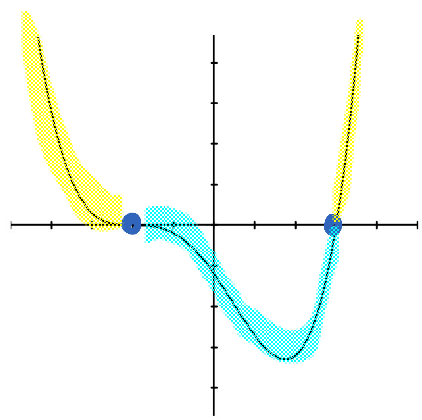
To determine the multiplicity of a zero/root from a graph, consider the following:



- Zeroes of **ODD** multiplicity change sign at the zero.
- Zeroes of **EVEN** multiplicity do not change sign at the zero.

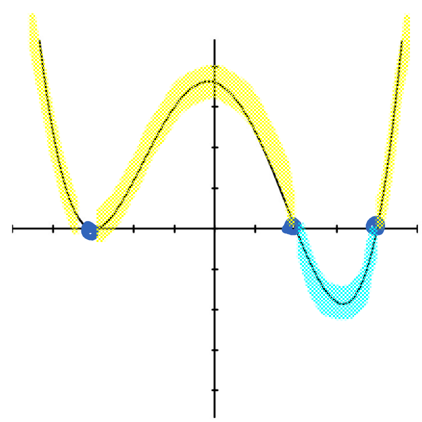
Example 1: For each graph, state the x-intercepts, the intervals where the function is positive and negative, whether the zeroes are of multiplicity 1,2, or 3.

a)



x-intercepts	$x=-2$ $x=3$ $(-2,0)$ $(3,0)$
Multiplicity	$x=-2$ Mult 3 $x=3$ Mult 1
Positive interval	$x<-2$ $x>3$
Negative interval	$-2<x<3$

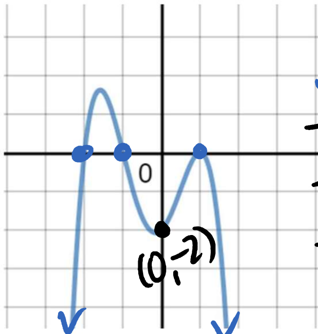
b)



x-intercepts	$x=-3$ $x=2$ $x=4$ $(-3,0)$ $(2,0)$ $(4,0)$
Multiplicity	$x=-3$ Mult 2 $\left.\begin{matrix}x=2\\x=4\end{matrix}\right\}$ Mult 1
Positive interval	$x<-3$ $-3<x<2$ $x>4$ $x<2$ $x\neq-3$ $x>4$
Negative interval	$2<x<4$

Example 2: For the following polynomial functions determine: the sign of the leading coefficient, the x-intercepts, multiplicity of the zeros, and an additional point. Use the information to find the equation of the polynomial function.

a)



$y = a(x+2)(x+1)(x-1)^2$

$-2 = a(0+2)(0+1)(0-1)^2$

$-2 = a(2)(1)(1)$

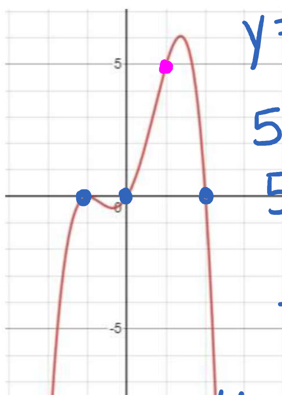
$-2 = 2a$

$-1 = a$

$y = -1(x+2)(x+1)(x-1)^2$

Sign of Leading Coefficient	Negative
x – intercepts	$x=-2$ $x=-1$ $x=1$
Multiplicity	$x=1$ Mult 2 $x=-2$ $x=-1$ } Mult 1
Additional Point	$(0,-2)$ x y

b)



$y = a(x+1)^2(x)(x-2)$

$5 = a(1+1)^2(1)(1-2)$

$5 = a(4)(1)(-1)$

$5 = -4a$

$-5/4 = a$

$y = -\frac{5}{4}x(x+1)^2(x-2)$

Sign of Leading Coefficient	$\ominus$
x – intercepts	$x=-1$ $x=0$ $x=2$
Multiplicity	$x=-1$ Mult 2 others are Mult 1
Additional Point	$(1,5)$

c) A degree 4 polynomial function has zeroes of -4, 1 (both multiplicity 1) and -2 (multiplicity 2). The constant term of the function is -3.

$y = a(x+4)(x-1)(x+2)^2$

$-3 = a(0+4)(0-1)(0+2)^2$

$-3 = a(4)(-1)(4)$

$-3 = -16a$

$3/16 = a$

$y = \frac{3}{16}(x+4)(x-1)(x+2)^2$

Sign of Leading Coefficient	
x – intercepts	$x=-4$ $x=1$ $x=-2$
Multiplicity	$x=-4$ $x=1$ Mult 1 / $x=-2$ Mult 2
Additional Point	$(0,-3)$

Practice: p.147 # 3, 4, 14 and worksheet