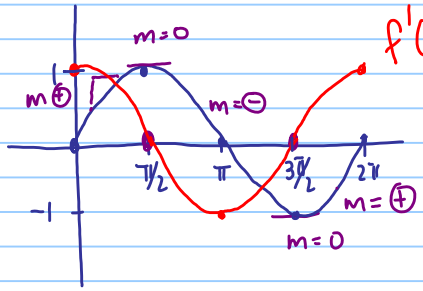
3.6 Trigonometric Functions

Note Title 10/22/2014

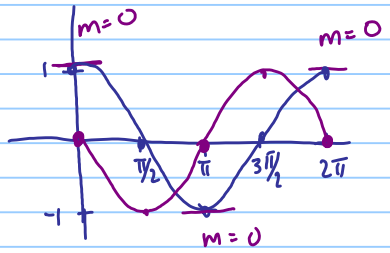
$f(x) = \sin x$



$f'(x) = \cos x$

$\frac{d}{dx} \sin x = \cos x$

$f(x) = \cos x$



$f'(x) = -\sin x$

$\frac{d}{dx} \cos x = -\sin x$

#1 $f(x) = x^2 \cos x \quad f'(x)$

product Rule

$f'(x) = x^2(-\sin x) + (\cos x)(2x)$

$= -x^2 \sin x + 2x \cos x$

#2 Find the equation for the tangent line at $\theta = 0$ if $g(\theta) = \frac{\theta}{\cos\theta}$

slope $= g'(0)$ point $(0, g(0))$

$(0,0)$

$g(0) = \frac{0}{\cos(0)} = \frac{0}{1} = 0$

$g'(\theta) = \frac{\cos\theta(1) - \theta(-\sin\theta)}{(\cos\theta)^2}$

$g'(\theta) = \frac{\cos\theta + \theta\sin\theta}{(\cos\theta)^2}$

$$g'(0) = \frac{\cos 0 + 0\sin 0}{(\cos 0)^2}$$

$$= \frac{1+0}{1^2}$$

$$= 1$$

$$y - y_1 = m(x - x_1)$$

$$y - g(0) = g'(0)(\theta - 0)$$

$$y - 0 = 1(\theta - 0)$$

$$y = \theta$$

Other Trig Derivatives

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

#3 show $\frac{d}{dx}\tan x = \sec^2 x$

$$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x} = \frac{\cos x(\cos x) - \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

#4 find $f'(x)$ if $f(x) = \frac{\csc^2 x}{x} = \frac{(\csc x)(\csc x)}{x}$ Product rule for top

$$f'(x) = \frac{x[\csc x(-\csc x \cot x) + \csc x(-\csc x \cot x)] - \csc^2 x(1)}{x^2}$$

$$= \frac{x[-2\csc^2 x \cot x] - \csc^2 x}{x^2}$$

$$= \frac{-\csc^2 x[2x\cot x + 1]}{x^2}$$

#5 $f(x) = \dfrac{\cot x}{3 - 3\sin x}$ find $f'(x)$

$$f'(x) = \frac{(3-3\sin x)(-\csc^2 x) - \cot x(-3\cos x)}{(3-3\sin x)^2}$$

$$= \frac{-3\csc^2 x + 3\sin x \csc^2 x + 3\cot x \cos x}{(3)^2(1-\sin x)^2}$$

$\sin x \cdot \frac{1}{\sin^2 x} = \frac{1}{\sin x}$

$$= \frac{-\csc^2 x + \sin x \csc^2 x + \cot x \cos x}{3(1-\sin x)^2}$$

$$= \frac{-\csc^2 x + \csc x + \cot x \cos x}{3(1-\sin x)^2}$$