# 3.7 The Chain Rule

Note Title 10/27/2014

## Composite Functions

Composite functions are found by plugging one function into the other

$$f(g(x)) = f \circ g$$

## The chain Rule:

If $f$ and $g$ are differentiable functions then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable

$$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$$

#1 $y = \sin(x^3)$ find $y'$

$f(u) = \sin u$ $\quad u = g(x) = x^3$

$f'(u) = \cos u$ $\quad g'(x) = 3x^2$

$$\frac{d}{dx}\sin(x^3) = \underbrace{\cos(x^3)}_{f'(g(x))} \cdot \underbrace{3x^2}_{g'(x)}$$

#2 $y = \sqrt{(x^2-1)^3}$ find $y'$

$y = (x^2-1)^{3/2}$

$f(u) = u^{3/2}$ $\quad u = g(x) = x^2-1$

$f'(u) = \frac{3}{2}u^{\frac{1}{2}}$ $\quad g'(x) = 2x$

$$y' = \frac{3}{2}(x^2-1)^{\frac{1}{2}} \cdot 2x$$

$$y' = 3x\sqrt{x^2-1}$$

## Other Notation

$$y = f(u) = f(g(x))$$

$$\frac{dy}{dx} = f'(u) \cdot g'(x)$$

$$= \frac{df}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

#3 $f(x) = (3x - 2x^2)^5$

$$y = u^5 \qquad u = 3x - 2x^2$$

$$\frac{dy}{dx} = 5u^4 \cdot (3 - 4x)$$

$$\frac{dy}{dx} = 5(3x - 2x^2)^4 (3 - 4x)$$

#4 $f(x) = \left(\frac{3x-1}{x^2+3}\right)^2$

$$y = u^2 \qquad u = \frac{3x-1}{x^2+3}$$

$$\frac{dy}{dx} = 2u \cdot \left[\frac{(x^2+3)(3) - (3x-1)(2x)}{(x^2+3)^2}\right]$$

$$= 2\left(\frac{3x-1}{x^2+3}\right)\left[\frac{3x^2+9-6x^2+2x}{(x^2+3)^2}\right]$$

$$= \frac{2(3x-1)(-3x^2+2x+9)}{(x^2+3)^3}$$

#5 $f(x) = x^2\sqrt{1-x^2}$

$= x^2(1-x^2)^{\frac{1}{2}}$

$f'(x) = x^2 \cdot \frac{d}{dx}(1-x^2)^{\frac{1}{2}} + 2x(1-x^2)^{\frac{1}{2}}$

$u = 1-x^2$

$y = u^{\frac{1}{2}}$

$f'(x) = x^2 \cdot \frac{1}{2}u^{-\frac{1}{2}} \cdot (-2x) + 2x(1-x^2)^{1/2}$

$= x^2 \cdot \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) + 2x(1-x^2)^{1/2}$

$= -x^3(1-x^2)^{-\frac{1}{2}} + 2x(1-x^2)^{1/2}$

$= x(1-x^2)^{-1/2}\left[-x^2(1) + 2(1-x^2)^1\right]$

$= x(1-x^2)^{-\frac{1}{2}}\left[-x^2+2-2x^2\right]$

$= \dfrac{x(-3x^2+2)}{\sqrt{1-x^2}}$

# Find the equation of the tangent line to $f(\theta) = \sin(\cos\theta)$ at $\theta = \frac{\pi}{2}$

$y = f(\frac{\pi}{2}) = \sin(\cos\frac{\pi}{2})$

$= \sin(0)$

$= 0$

$(\frac{\pi}{2}, 0)$

$m = f'(\frac{\pi}{2})$

$y = \sin u$

$u = \cos\theta$

$f'(\theta) = \cos u \cdot (-\sin\theta)$

$= \cos(\cos\theta)(-\sin\theta)$

$f'(\pi/2) = \cos(\cos\frac{\pi}{2})(-\sin\frac{\pi}{2})$

$= \cos(0)(-1)$

$= (1)(-1)$

$= -1$

$y - 0 = -1(x - \frac{\pi}{2})$

$y = -x + \frac{\pi}{2}$