# 3.8 Implicit Differentiation

Note Title 10/31/2014

$y^4 + xy = x^3 - x + 2$

can't be rewritten as $y =$ (explicit form)

Eg $x^2 + y^2 = 1$ (Implicit form)

find $\frac{dy}{dx}$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$$

$$2x + \boxed{\frac{d}{dx}(y^2)} = 0$$

$y = f(x)$

$y^2 = (f(x))^2$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(f(x))^2 = 2[f(x)]^1 \cdot \frac{df}{dx} = 2f(x)\frac{dy}{dx}$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

#1 Find $\frac{dy}{dx}$ $\quad y^3+y^2-5y-x^2=-4$

$$\frac{d}{dx}\left(y^3+y^2-5y-x^2\right)=\frac{d}{dx}(-4)$$

$$3y^2\cdot\frac{dy}{dx}+2y\cdot\frac{dy}{dx}-5\cdot\frac{dy}{dx}-2x=0$$

$$\frac{dy}{dx}\left(3y^2+2y-5\right)=2x$$

$$\frac{dy}{dx}=\frac{2x}{3y^2+2y-5}$$

#2 Find $\frac{dy}{dx}$ $\quad x^2(x^2+y^2)=y^2$ at $\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

$$\frac{d}{dx}\left(x^2(x^2+y^2)\right)=\frac{d}{dx}\left(y^2\right)$$

product

$$x^2\cdot\left(2x+2y\frac{dy}{dx}\right)+2x\left(x^2+y^2\right)=2y\frac{dy}{dx}$$

$$2x^3+2x^2y\frac{dy}{dx}+2x^3+2xy^2=2y\frac{dy}{dx}$$

$$4x^3+2xy^2=\frac{dy}{dx}\left(2y-2x^2y\right)$$

$$\frac{dy}{dx}=\frac{4x^3+2xy^2}{2y-2x^2y}$$

$$\frac{dy}{dx}=\frac{x\left(2x^2+y^2\right)}{y\left(1-x^2\right)}$$

$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

$$\frac{dy}{dx}\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)=\frac{\cancel{\frac{\sqrt{2}}{2}}\left(2\left(\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2\right)}{\cancel{\frac{\sqrt{2}}{2}}\left(1-\left(\frac{\sqrt{2}}{2}\right)^2\right)}$$

$$=\frac{2\left(\frac{2}{4}\right)+\frac{2}{4}}{1-\frac{2}{4}}=\frac{\frac{4}{4}+\frac{2}{4}}{\frac{4}{4}-\frac{2}{4}}=\frac{\frac{6}{4}}{\frac{2}{4}}=3$$

#3 $x\sin y - y\cos x = 2$ find $\frac{dy}{dx}$

$$\frac{d}{dx}\left(x\sin y - y\cos x\right) = \frac{d}{dx}(2)$$

$$x\cdot(\cos y)\cdot\frac{dy}{dx} + 1\cdot\sin y - \left[y\cdot(-\sin x) + 1\frac{dy}{dx}\cos x\right] = 0$$

$$x(\cos y)\frac{dy}{dx} - (\cos x)\frac{dy}{dx} = -\sin y - y\sin x$$

$$\frac{dy}{dx}\left(x\cos y - \cos x\right) = -\sin y - y\sin x$$

$$\frac{dy}{dx} = \frac{-\sin y - y\sin x}{x\cos y - \cos x}$$

or

$$\frac{dy}{dx} = \frac{\sin y + y\sin x}{\cos x - x\cos y}$$

#4 Find the equation of the tangent line at $(2,1)$ of $x^2y^3 + 2y = 3x$

① Find $\frac{dy}{dx}$

② $m = \frac{dy}{dx}$ at a point

③ $y - y_1 = m(x - x_1)$

$$\frac{d}{dx}\left(x^2y^3 + 2y\right) = \frac{d}{dx}(3x)$$

$$x^2\cdot 3y^2\frac{dy}{dx} + 2x\cdot y^3 + 2\frac{dy}{dx} = 3$$

$$\frac{dy}{dx}\left(3x^2y^2 + 2\right) = 3 - 2xy^3$$

$$\frac{dy}{dx} = \frac{3 - 2xy^3}{3x^2y^2 + 2}$$

Find $m$ = slope

Evaluate $\frac{dy}{dx}$ at $(2,1)$

$$\frac{dy}{dx}(2,1) = \frac{3-2(2)(1)^3}{3(2)^2(1)^2+2}$$

$$= \frac{3-4}{12+2}$$

$$m = \frac{-1}{14}$$

$m = -\frac{1}{14}$ $(2,1)$

$$y-1 = -\frac{1}{14}(x-2)$$

$$y = -\frac{1}{14}x + \frac{2}{14} + 1$$

$$y = -\frac{1}{14}x + \frac{8}{7}$$