4.1 Angles and Angle Measure

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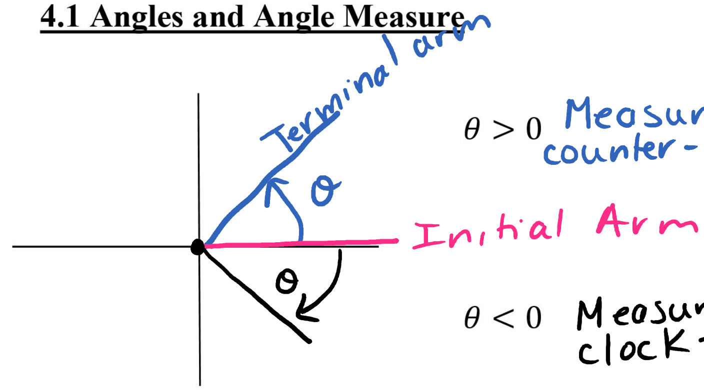


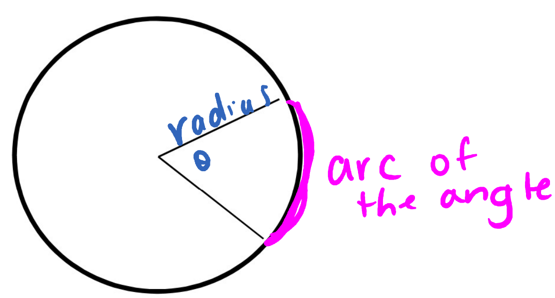
$\theta > 0$ Measured counter-clockwise

$\theta < 0$ Measured clock-wise

Angles in standard position have their center at the origin ___(0,0)___ and the ___initial arm___ on the positive x-axis.

Angles are measured in ___degree___ or ___radians___.

An angle that has a measure of 1 radian is an angle in which the length of the ___radius___ = length of ___the arc of the angle___



	Degrees	Radian Measure
Full rotation of a circle	360°	2π
Half rotation of a circle	180°	π
¼ rotation of a circle	90°	$\pi/2$

Note: Angles without units are considered radians.

Conversion factor degrees and radians:

$$\pi \, radians = 180°$$

degrees → radians

$$\text{degrees}\left(\frac{\pi}{180°}\right) = \text{radians}$$

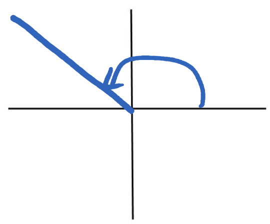
radians → degrees

$$\text{radians}\left(\frac{180°}{\pi}\right) = \text{degrees}$$

Ex. #1: Convert each angle into degrees or radian. Draw each angle in standard position

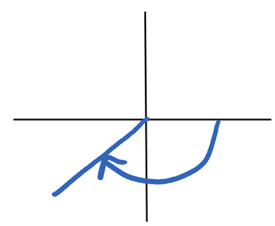
(a) 162°

$$162°\left(\frac{\pi}{180°}\right) = \frac{162\pi}{180} = \frac{81\pi}{90} = \frac{9\pi}{10}$$

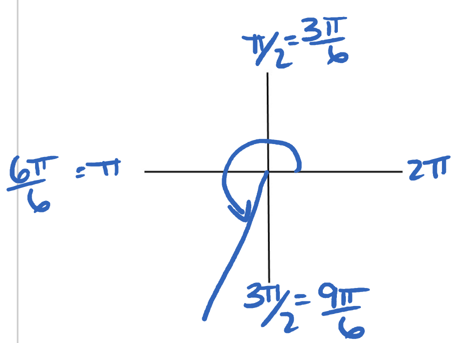


(b) $-150°$

$$-150°\left(\frac{\pi}{180°}\right) = \frac{-150\pi}{180} = \frac{-15\pi}{18} = -\frac{5\pi}{6}$$



(c) $\frac{4\pi}{3} = \frac{8\pi}{6}$



$$\frac{4\cancel{\pi}}{3}\left(\frac{180°}{\cancel{\pi}}\right)$$

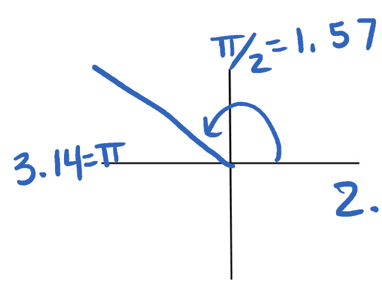
$$\frac{4(\cancel{180}^{60}°)}{\cancel{3}}$$

$$4(60°)$$

$$240°$$

(d) 2.46 radians

use calculator

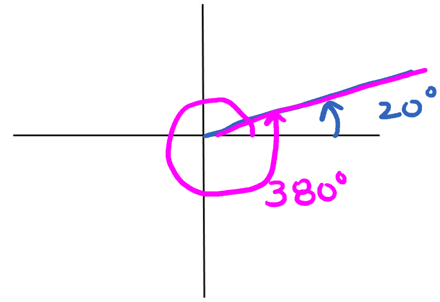


$$2.46\left(\frac{180°}{\pi}\right)$$

$$= 140.95°$$

Coterminal Angles: Angles that have the same terminal arm when in standard position.

Draw an angle of 20° and one of 380°.



What do you notice about the two angles? same terminal arm

We can find coterminal angles by Add 360° or 2π to the angle

Ex. #2: Find two coterminal angles for each given angle. Express your answer in general form.

(a) 150°

$\theta = 150° + 360°$

$\theta = 510°$

$\theta = 150° - 360°$

$\theta = -210°$

$\theta = 150° + 360°n \quad n \in I \quad n \neq 0$

or $\theta = 150° \pm 360°n \quad n \in N$

(b) $\frac{\pi}{3}$

$\theta = \frac{\pi}{3} + 2\pi$

$\theta = \frac{\pi}{3} + \frac{6\pi}{3}$

$\theta = \frac{7\pi}{3}$

$\theta = \frac{\pi}{3} - 2\pi$

$\theta = \frac{\pi}{3} - \frac{6\pi}{3}$

$\theta = -\frac{5\pi}{3}$

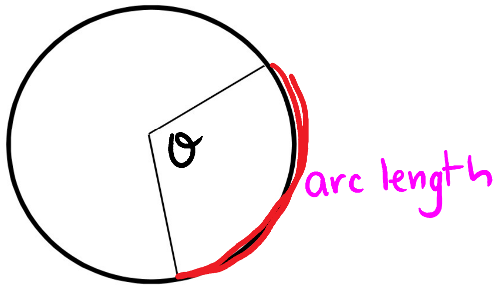
$\theta = \frac{\pi}{3} + 2\pi n \quad n \in I \quad n \neq 0$

$\theta = \frac{\pi}{3} \pm 2\pi n \quad n \in N$

Use your calculator

Arc Length of a Circle:

Arc Length is the length of the arc that subtends the central angle.



$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{full rotation}}$$

$2\pi r$ 360° or 2π

Arc length in degrees

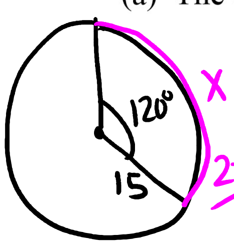
$$\frac{\text{arc length}}{2\pi r} = \frac{\theta}{360^\circ}$$

Arc length in radians

$$\frac{\text{arc length}}{2\pi r} = \frac{\theta}{2\pi}$$

Ex. #3: Find the arc length of the sector that is formed if

(a) The central angle is 120° and the radius of the circle is 15cm.



$$\frac{x}{2\pi(15)} = \frac{120^\circ}{360^\circ}$$

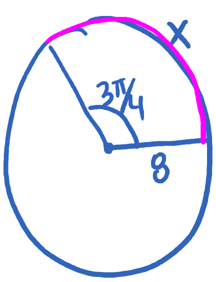
$$\cancel{2\pi(15)}\left(\frac{x}{\cancel{2\pi(15)}}\right) = \frac{120^\circ}{360^\circ}(2\pi)(15)$$

$$x = \frac{120^\circ}{360^\circ} \cdot 2\pi(15)$$

$$x = \frac{120^\circ}{360^\circ} \cdot 30\pi = 31.4\text{cm}$$

$$x = \frac{\cancel{12}^{1}}{\cancel{36}_{3}} \cdot \cancel{30}^{10}\pi = 10\pi$$

(b) The central angle is $\frac{3\pi}{4}$ and the radius of the circle is 8 units.



$$\frac{x}{2\pi(8)} = \frac{3\pi/4}{2\pi}$$

$$\cancel{2\pi(8)}\left(\frac{x}{\cancel{2\pi(8)}}\right) = \frac{3\pi/4}{\cancel{2\pi}}(\cancel{2\pi})(8)$$

$$x = \frac{3\pi}{4} \cdot 8$$

$x = 6\pi$ $\quad x = 18.85$ units