4.2 Extreme Values

Monday, June 14, 2021 10:42 AM

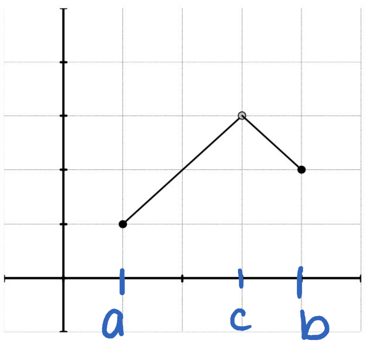
4.2 Extreme Values

Extreme Values on an Interval: Let $f(x)$ be a function on an interval, let "a" be on the interval

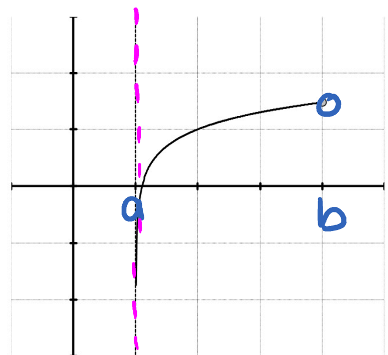
y-value

Absoulte Minimum on $f(x)$ is $f(a)$ if $f(a) \leq f(x)$ for all x on the interval
Absoulte Maximum on $f(x)$ is $f(a)$ if $f(a) \geq f(x)$ for all x on the interval

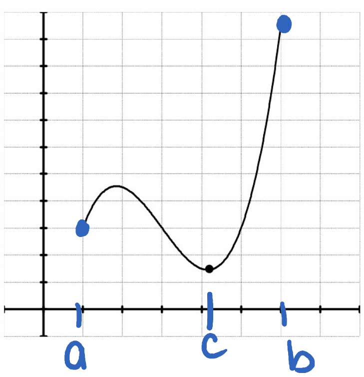
y-value



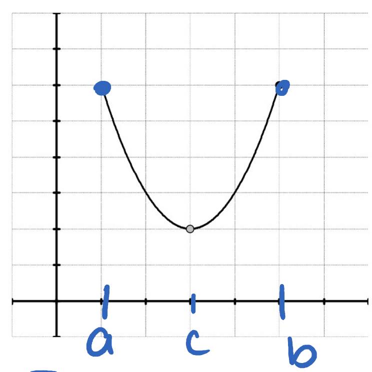
[a,b] f(a) Minimum
No Maximum
Not continuous at x = c



(a, b) No Max
No Min
Not defined at x = a or x = b



[a,b] Max f(b)
Min f(c)



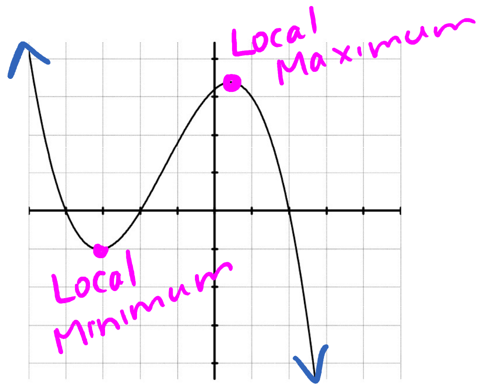
[a, b] Max f(a) = f(b)
No Min
Not continuous at x = c

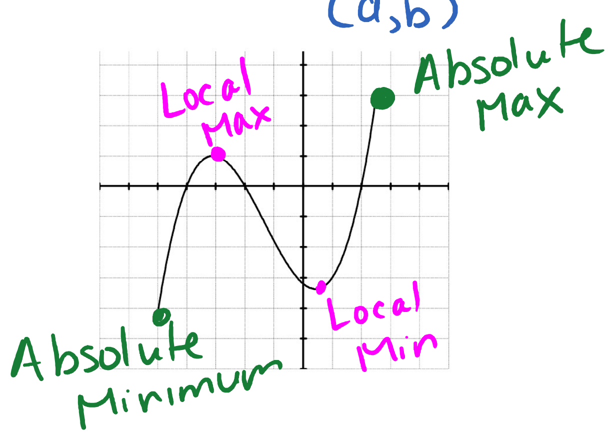
<u>Existence of Extrema:</u> A continuous function *f* on a closed interval [a,b] has both a maximum and a minimum.

<u>Local Extrema: (Relative Extrema)</u>

Local Minimum: at *x=c* if *f(c)* is the minimum on an open interval (a,b)

Local Maximum: at *x=c* if *f(c)* is the maximum on an open interval (a,b)





<u>Critical Points:</u>

A number "c" in the domain of f is called a critical point if either:

$f'(c) = 0$ or $f'(c)$ is undefined

Local extrema occur at critical points

<u>Absolute extrema occur at endpoints or at critical points.</u>

1. Find the absolute extrema of $f(x) = 3x^4 - 4x^3$ on $[-1,2]$

$f'(x) = 12x^3 - 12x^2$

$0 = 12x^3 - 12x^2$

$0 = 12x^2(x-1)$

$x = 0 \qquad x = 1$ critical numbers

$f(x) = 3x^4 - 4x^3$

	Value of x	Value of f
critical numbers	$x = 0$	$f(0) = 0$
	$x = 1$	$f(1) = 3 - 4 = -1$
endpoints	$x = -1$	$f(-1) = 3(-1)^4 - 4(-1)^3 = 3 + 4 = 7$
	$x = 2$	$f(2) = 3(2)^4 - 4(2)^3 = 48 - 32 = 16$

Absolute Max $f(2) = 16$

Absolute Min $f(1) = -1$

2. Find the absolute extrema of $f(x) = 2x - 3x^{\frac{2}{3}}$ on $[-1,3]$

$f'(x) = 2 - \cancel{3}\left(\frac{2}{\cancel{3}}\right)x^{-1/3}$

$f'(x) = 2 - \dfrac{2}{x^{1/3}}$

$f'(x) = \dfrac{2x^{1/3}}{x^{1/3}} - \dfrac{2}{x^{1/3}}$

$f'(x) = \dfrac{2x^{1/3} - 2}{x^{1/3}}$

$f'(x) = 0$

Numerator $= 0$

$2x^{1/3} - 2 = 0$

$x^{1/3} - 1 = 0$

$(x^{1/3})^3 = 1^3$

$x = 1$

$f'(x)$ undefined

Denominator $= 0$

$x^{1/3} = 0$

$x = 0$

AP Calculus

$f(x) = 2x - 3x^{2/3}$

	Value of x	Value of f
critical numbers	$x = 1$	$f(1) = 2 - 3$ $f(1) = -1$
	$x = 0$	$f(0) = 0 - 0$ $f(0) = 0$
endpoints	$x = -1$	$f(-1) = 2(-1) - 3(-1)^{2/3}$ $= -2 - 3$ $= -5$
	$x = 3$	$f(3) = 2(3) - 3(3)^{2/3}$ $= -0.24$

Absolute Max $f(0) = 0$

Absolute Min $f(-1) = -5$

3. Find the absolute extrema of $f(x) = 2\sin x - \cos 2x$ on $[0, 2\pi]$

$f'(x) = 2\cos x - (-\sin 2x) \cdot 2$

$f'(x) = 2\cos x + 2\sin 2x$

$0 = 2\cos x + 2\sin 2x$ $\quad$ $\sin 2x = 2\sin x \cos x$

$0 = 2\cos x + 2(2\sin x \cos x)$

$0 = 2\cos x + 4\sin x \cos x$

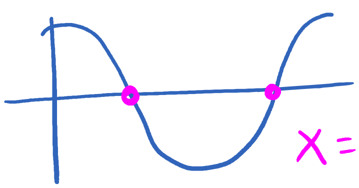
$0 = 2\cos x(1 + 2\sin x)$

$2\cos x = 0$

$\cos x = 0$

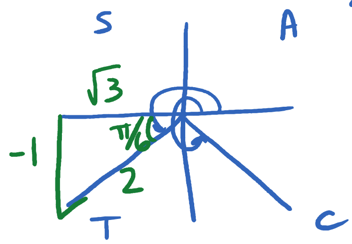
$1 + 2\sin x = 0$

$\sin x = -\frac{1}{2}$



$x = \pi/2$

$x = 3\pi/2$



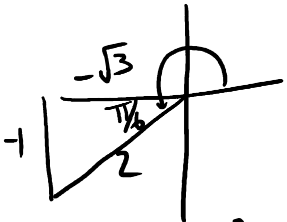
ref ∠ = $\pi/6$

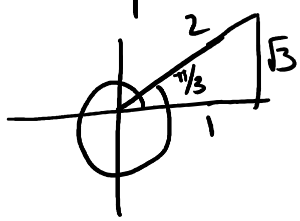
$\theta = \pi + \pi/6$

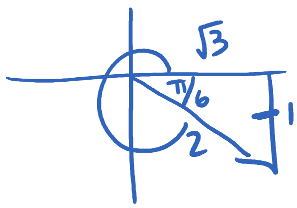
$\theta = 7\pi/6$

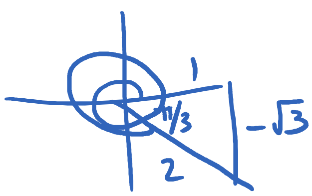
$\theta = 2\pi - \pi/6$

$\theta = \frac{11\pi}{6}$









Value of x	Value of f
$x=\pi/2$	$f(\pi/2) = 2\sin \pi/2 - \cos(2\cdot\pi/2)$ $= 2\sin\pi/2 - \cos\pi$ $= 2(1) - (-1) = 3$
$x=3\pi/2$	$f(3\pi/2) = 2\sin 3\pi/2 - \cos(2\cdot 3\pi/2)$ $= 2\sin 3\pi/2 - \cos(3\pi)$ $= 2(-1) - (-1) = -1$
$x=7\pi/6$	$f(7\pi/6) = 2\sin 7\pi/6 - \cos(2\cdot 7\pi/6)$ $= 2\sin 7\pi/6 - \cos 7\pi/3$ $= 2(-1/2) - \frac{1}{2} = -1 - 1/2 = -3/2$
$x=11\pi/6$	$f(11\pi/6) = 2\sin \frac{11\pi}{6} - \cos(2\cdot 11\pi/6)$ $= 2\sin 11\pi/6 - \cos 11\pi/3$ $= 2(\frac{-1}{2}) - (\frac{1}{2})$ $= -1 - 1/2 = -3/2$
$x=0$	$f(0) = 2\sin(0) - \cos(2(0))$ $= 2(0) - (1)$ $= -1$
$x=2\pi$	$f(2\pi) = -1$
Absolute Max $f(\pi/2)=3$	Absolute Min $f(7\pi/6) = f(11\pi/6) = -\frac{3}{2}$

Rolle's Theorem:
$f(x)$ is continuous on [a,b] and differentiable on (a,b).
If $f(a) = f(b)$ then there is a number "c" such that $f'(c) = 0$

4. Find all values of c on [-2,2] such that $f'(c) = 0$ $f(x) = x^4 - 2x^2$

f(x) is continuous and differentiable

$a = -2$
$f(-2) = (-2)^4 - 2(-2)^2$
$= 16 - 8$
$= 8$

$b = 2$
$f(2) = (2)^4 - 2(2)^2$
$= 16 - 8$
$= 8$

$f(-2) = f(2)$

$\therefore$ There exists a point c where $f'(c) = 0$

$f'(x) = 4x^3 - 4x$
$0 = 4x^3 - 4x$
$0 = 4x(x^2 - 1)$
$x = 0 \quad x = 1 \quad x = -1$