4.2 The Unit Circle

Wednesday, February 22, 2017 9:05 AM

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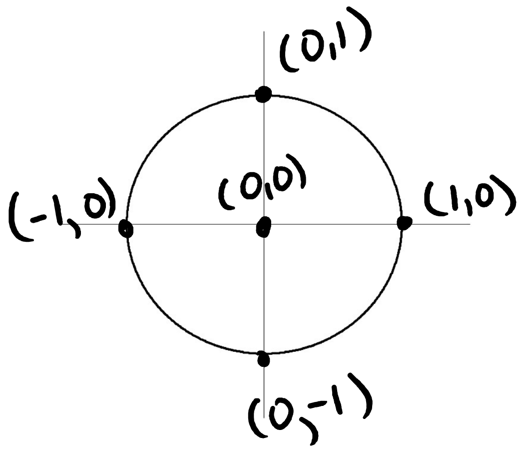


Unit circle:

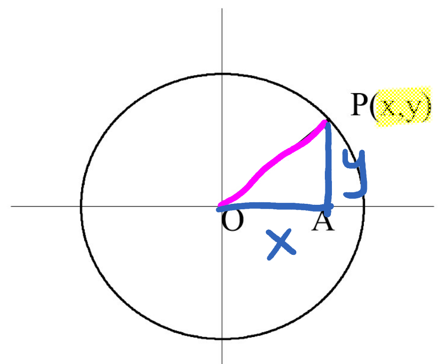
A circle with a radius of 1

and its center is at the origin

OP = radius

OA = x

PA = y



On the unit circle the radius is equal to one. The Pythagorean Theorem can be applied to the triangle. $a^2 + b^2 = c^2$

The equation of a circle is $x^2 + y^2 = r^2$. Therefore the equation of the unit circle is: $x^2 + y^2 = 1$

Ex. #1: Determine the equation of a circle with radius of 5 and the center at the origin.

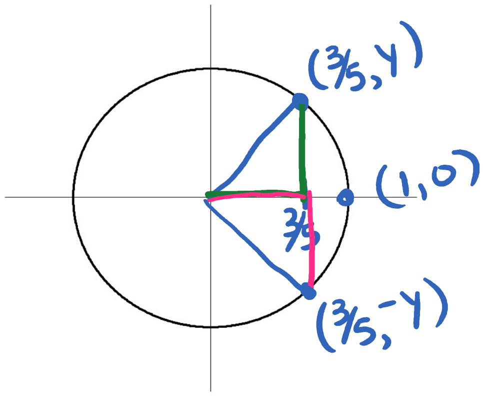
$x^2 + y^2 = r^2$ $\quad r = 5$

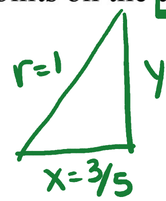
$x^2 + y^2 = 5^2$

$x^2 + y^2 = 25$

Ex. #2: Determine the coordinates of all points on the $\boxed{\text{unit circle}}$ with an:

(a) x-coordinate of x = 3/5



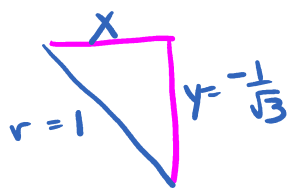


$x^2 + y^2 = 1$
$\left(\frac{3}{5}\right)^2 + y^2 = 1$
$\frac{9}{25} + y^2 = 1$
$y^2 = \frac{25}{25} - \frac{9}{25}$
$y^2 = \frac{16}{25}$
$y = \pm\frac{4}{5}$

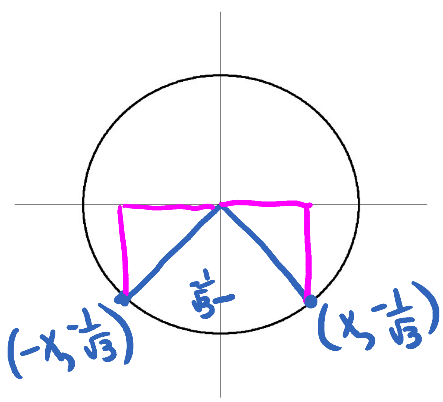
Quad 1
$\left(\frac{3}{5}, \frac{4}{5}\right)$

Quad 4
$\left(\frac{3}{5}, -\frac{4}{5}\right)$

(b) y-coordinate of $y = -\frac{1}{\sqrt{3}}$



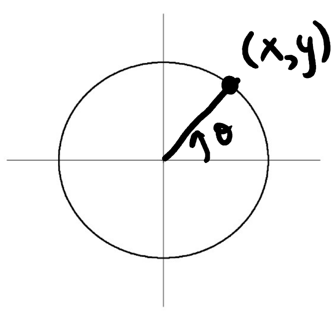
$x^2 + y^2 = 1$
$x^2 + \left(-\frac{1}{\sqrt{3}}\right)^2 = 1$
$x^2 + \frac{1}{3} = 1$
$x^2 = \frac{3}{3} - \frac{1}{3}$
$x^2 = \frac{2}{3}$
$x = \pm\frac{\sqrt{2}}{\sqrt{3}}$



Quad 3
$\left(-\frac{\sqrt{2}}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

Quad 4
$\left(\frac{\sqrt{2}}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

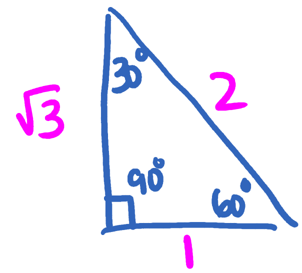
__$P(\theta)$__ refers to the point P on the unit circle that forms the angle θ.

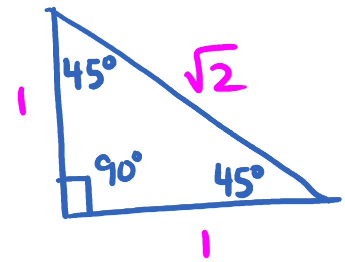


Since P is a point it has the coordinates:

$P(\theta) = (x, y)$

Special Triangles in degrees



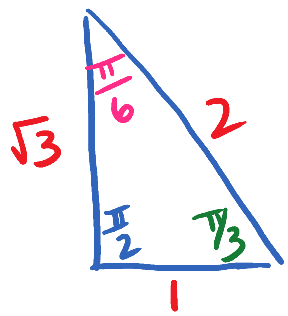


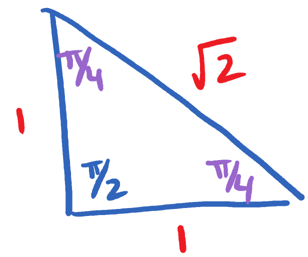
$\cancel{30}^{\,1}\left(\frac{\pi}{\cancel{180}_{6}}\right) = \frac{\pi}{6}$

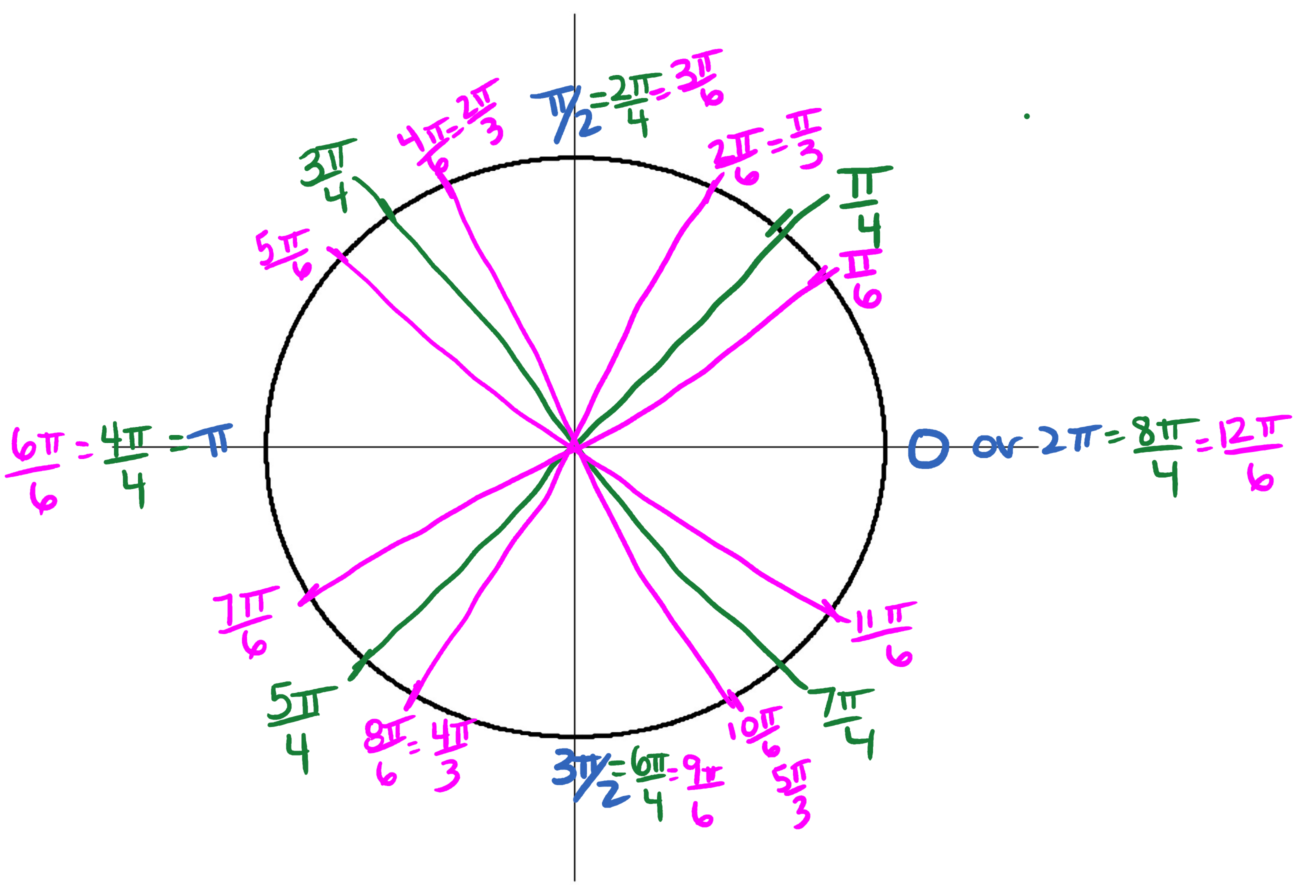
$\cancel{60}^{\,1}\left(\frac{\pi}{\cancel{180}_{3}}\right) = \frac{\pi}{3}$

$\cancel{45}^{\,1}\left(\frac{\pi}{\cancel{180}_{4}}\right) = \frac{\pi}{4}$

Special Triangles in radians ~~on the unit circle~~





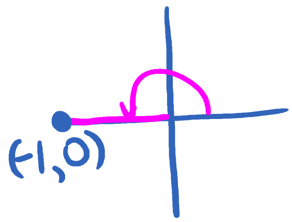


$P(\theta) = (x, y)$ on the unit circle

Ex. #3: Find the coordinates of :

(a) $P(\pi)$

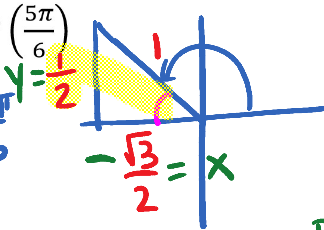
$\theta = \pi$



$P(\pi) = (-1, 0)$

(b) $P\left(\frac{5\pi}{6}\right)$

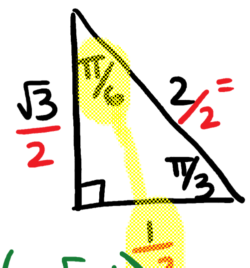
$\theta = \frac{5\pi}{6}$



ref $\angle = \pi - \frac{5\pi}{6}$

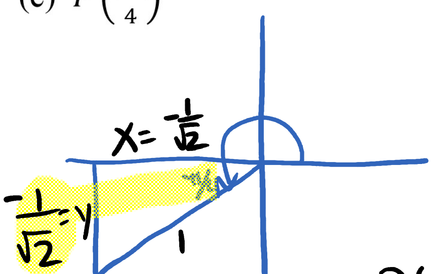
ref $\angle = \frac{6\pi}{6} - \frac{5\pi}{6}$

ref $= \frac{\pi}{6}$



$P(\theta) = (x, y)$ $\quad P\left(\frac{5\pi}{6}\right)\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

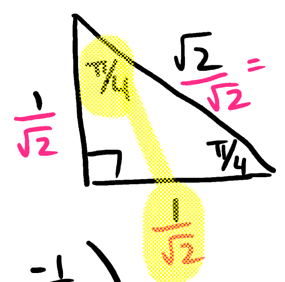
(c) $P\left(\frac{5\pi}{4}\right)$



ref $\angle = \frac{5\pi}{4} - \pi$

ref $\angle = \frac{5\pi}{4} - \frac{4\pi}{4}$

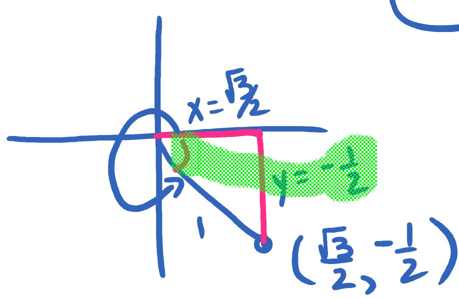
ref $\angle = \frac{\pi}{4}$

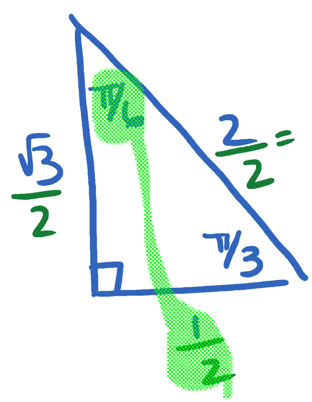


$P(\theta) = (x, y)$ $\quad P\left(\frac{5\pi}{4}\right) = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

Ex. #4: Find angle θ such that $P(\theta) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$r = 1$





ref $\angle = \frac{\pi}{6}$

$\theta = 2\pi - \frac{\pi}{6}$

$\theta = \frac{12\pi}{6} - \frac{\pi}{6}$

$\theta = \frac{11\pi}{6}$