4.3 Trigonometric Ratios

Monday, October 18, 2021 10:56 AM

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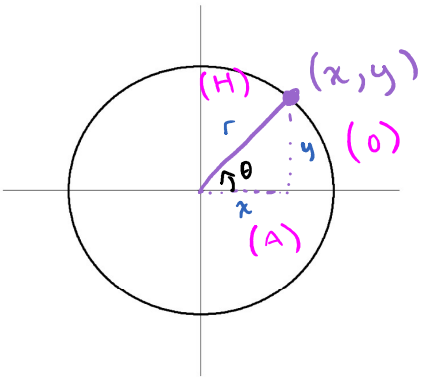
If (x, y) is a point on the terminal arm of a circle then the following ratios can be determined:



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

Reciprocal Trig Ratios

Cosecant is the reciprocal of sine

$\csc\theta = \frac{1}{\sin\theta}$ or $\csc\theta = \frac{1}{\left(\frac{y}{r}\right)} = \frac{r}{y}$

Secant is the reciprocal of cosine

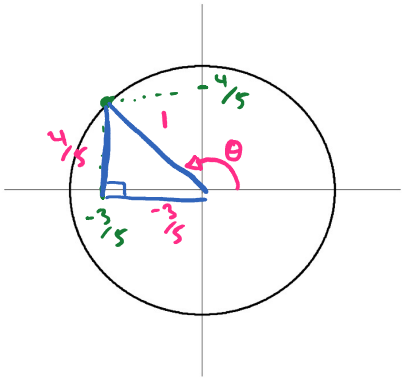
$\sec\theta = \frac{1}{\cos\theta}$ or $\sec\theta = \frac{r}{x}$

Cotangent is the reciprocal of tangent

$\cot\theta = \frac{1}{\tan\theta}$ or $\cot\theta = \frac{x}{y}$

$r = 1$

Ex. #1: The point $\left(-\frac{3}{5}, \frac{4}{5}\right)$ is on the **unit circle** and the terminal arm of angle θ. Find the value of all six trig ratios.



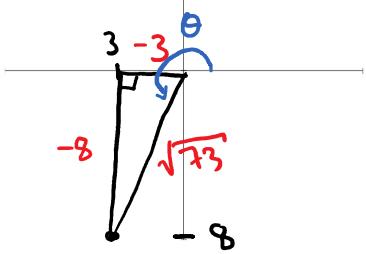
$\sin\theta = \frac{y}{r} = \frac{(4/5)}{1} = \frac{4}{5}$

$\cos\theta = \frac{x}{r} = \frac{(-3/5)}{1} = \frac{-3}{5}$

$\tan\theta = \frac{y}{x} = \frac{(4/5)}{(-3/5)} = \frac{4(5)}{-3(5)} = \frac{-4}{3}$

$\csc\theta = \frac{5}{4}$ $\quad \sec\theta = \frac{-5}{3}$ $\quad \cot\theta = -\frac{3}{4}$

Ex. #2: The point $(-3, -8)$ is on the terminal arm of angle θ. Find the value of all six trig ratios.



$x^2 + y^2 = r^2$

$(-3)^2 + (-8)^2 = r^2$

$9 + 64 = r^2$

$73 = r^2$

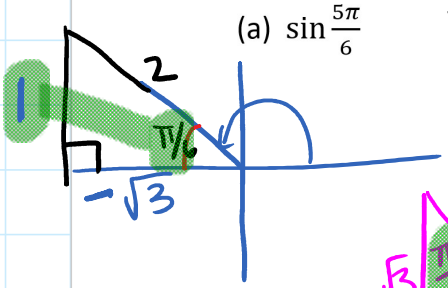
$r = \sqrt{73}$

$\sin\theta = \frac{y}{r} = \frac{-8}{\sqrt{73}}$ $\quad \cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{73}}$ $\quad \tan\theta = \frac{y}{x} = \frac{-8}{-3} = \frac{8}{3}$

$\csc\theta = \frac{-\sqrt{73}}{8}$ $\quad \sec\theta = \frac{-\sqrt{73}}{3}$ $\quad \cot\theta = \frac{3}{8}$

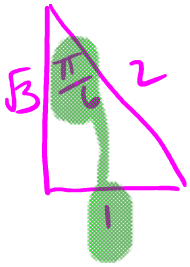
Ex. #3: Find the exact trig ratio:

(a) $\sin\frac{5\pi}{6}$



$$\text{ref} = \pi - \frac{5\pi}{6}$$
$$= \frac{6\pi}{6} - \frac{5\pi}{6}$$
$$= \pi/6$$

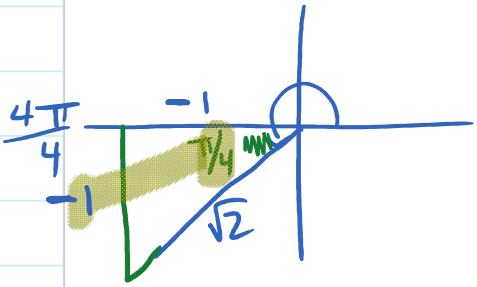
Draw the angle
Find reference angle
Apply special Δ



$$\sin\frac{5\pi}{6} = \frac{1}{2}$$

$$\sin\theta = \frac{y}{r}$$

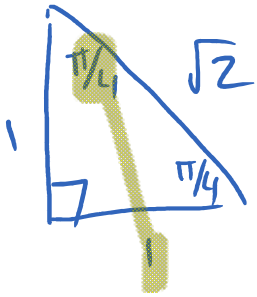
(b) $\cos\frac{5\pi}{4}$



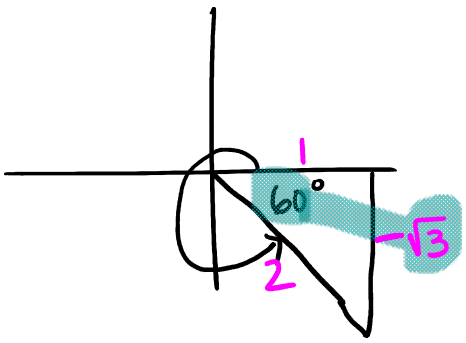
$$\text{ref} = \frac{5\pi}{4} - \frac{4\pi}{4}$$
$$\text{ref} = \frac{\pi}{4}$$

$$\cos\theta = \frac{x}{r}$$

$$\cos\frac{5\pi}{4} = \frac{-1}{\sqrt{2}}$$



(c) $\csc 300°$

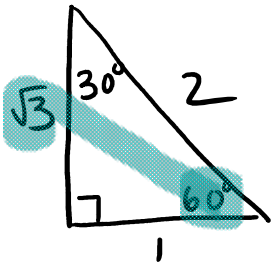


$$\text{ref}\angle = 360° - 300°$$
$$= 60°$$

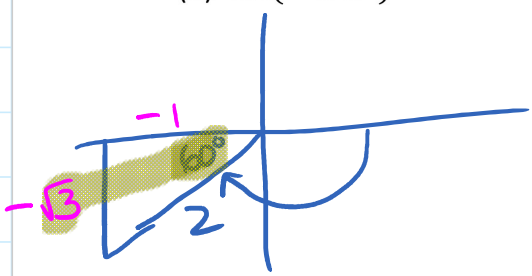
$$\sin\theta = \frac{y}{r}$$

$$\sin 300° = -\frac{\sqrt{3}}{2}$$

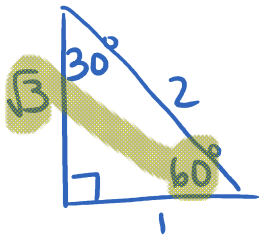
$$\csc 300° = -\frac{2}{\sqrt{3}}$$



(d) $\cot(-120°)$



ref = 60°



$\tan\theta = \frac{y}{x}$

$\tan(-120°) = \frac{-\sqrt{3}}{-1}$

$\cot(-120°) = \frac{1}{\sqrt{3}}$

Ex. #4: Use a calculator to find the trig ratios (to the nearest thousandth)

(a) $\sin 50°$

(b) $\sec 100°$

(c) $\tan\frac{5\pi}{7}$