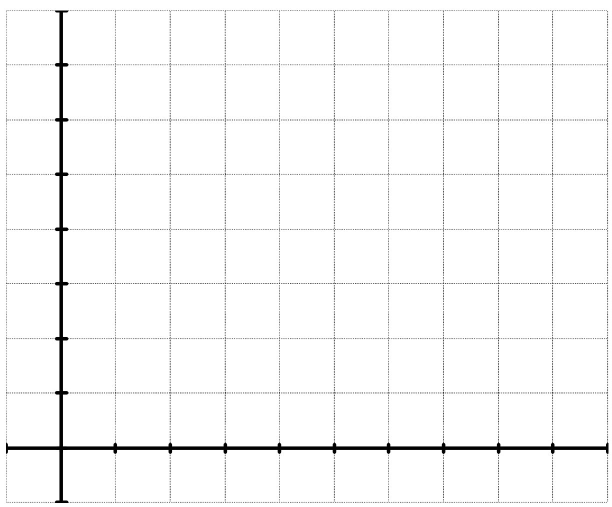
4.3 Mean Value Theorem and Monotonicity

Mean Value Theorem: $f(x)$ is continuous on [a,b] and differentiable on (a,b). Then there exists at least one value *c* on (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



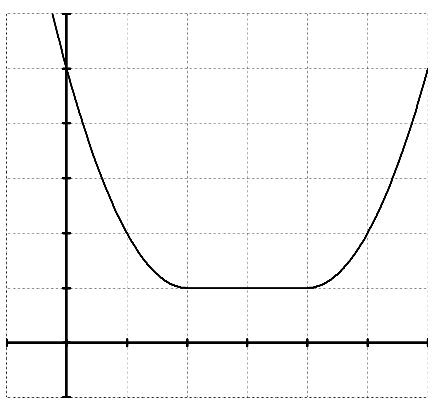
Slope of Secant Line =

Slope of Tangent Line =

The Mean Value Theorem implies that:

1. Verify the Mean Value Theorem (MVT) for $f(x) = 5 - \frac{4}{x}$ on the interval (1,4).

Increasing and Decreasing Behavior of Functions

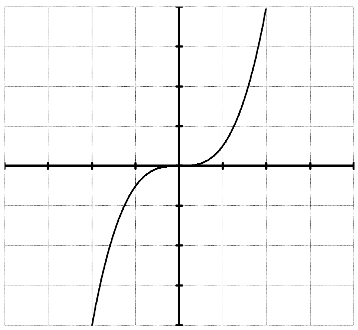


Increasing on (a,b)

Decreasing on (a,b)

Monotonic

f(x) is montonic on (a,b) if the function is always increasing or always decreasing



2. Find the intervals where f(x) is increasing and decreasing. $f(x) = x^4 - 4x^3 - 8x^2 - 1$

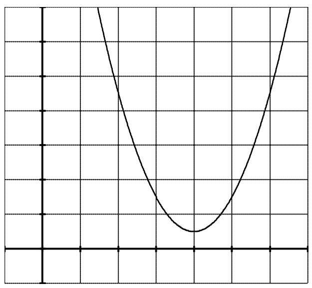
<u>First Derivative Test</u> <u>For Critical Points:</u>

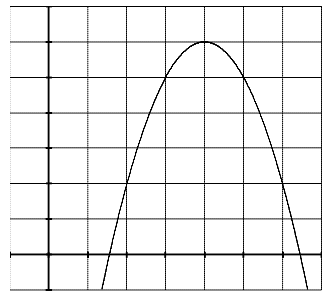
f(x) is differentiable and 'c' is a critical point of the function.

Critical points occur when $f'(x) = 0 \quad or \quad f'(x) is\ undefined$

If $f'(x)$ changes from positive to negative at c, then $f(c)$ is a local maximum

If $f'(x)$ changes from negative to positive at c, then $f(c)$ is a local minimum





3. Find the extrema of $f(x) = \frac{1}{2}x - \sin x$ on $(0, 2\pi)$

4. Find the relative extrema of $f(x) = \frac{x^4+1}{x^2}$