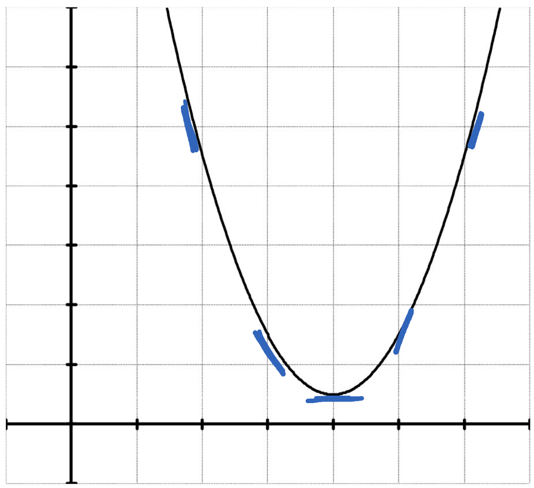
4.4 Shape of a Graph Part 1

Monday, June 14, 2021 11:04 AM

4.4 The Shape Of A Graph Part 1

Concavity: The graph of f is concave upwards if f' is increasing and concave downwards if f' is decreasing.

Concave Up



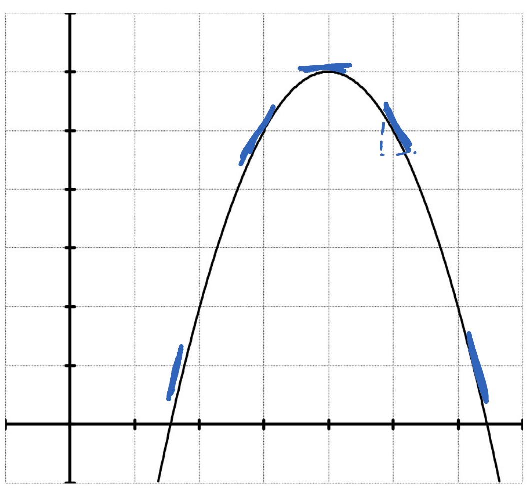
Makes a cup

left → right

$f'(x)$ is increasing

Tangent lines below the graph of $f(x)$

Concave Down



Makes a frown

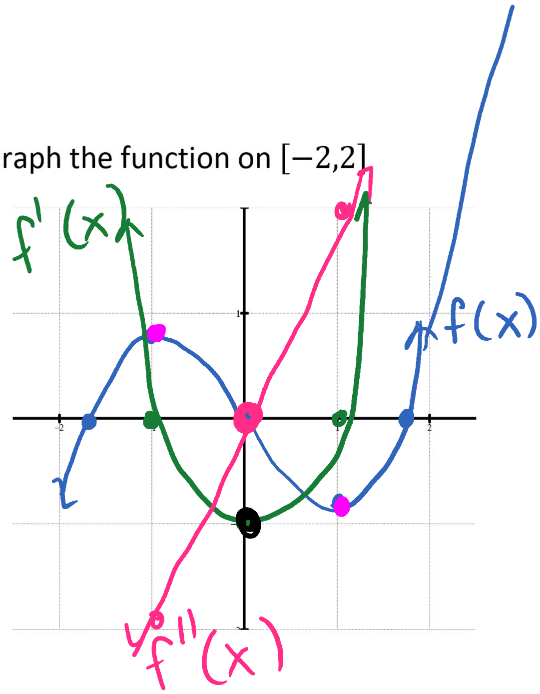
left → right

$f'(x)$ is decreasing

Tangent lines above the graph of $f(x)$

AP Calculus

#1 $f(x) = \frac{1}{3}x^3 - x$ Graph the function on $[-2,2]$



a) Concavity of the function

concave up $x > 0$

concave down $x < 0$

b) Relationship of the zeroes of $f'(x)$ and the function

$f'(x) = x^2 - 1$

$0 = x^2 - 1$

$1 = x^2$

$x = \pm 1$

when $x = 1$ $x = -1$ f(x) Has a max or a min

c) Relationship of the zeroes of $f''(x)$ and the function

$f''(x) = 2x$

$0 = 2x$

$0 = x$

when $x = 0$ f(x) changes concavity

d) Relationship of the zeroes of $f''(x)$ and the first derivative

zeros of $f''(x)$

$x = 0$

when $x = 0$ $f'(x)$ has a minimum

Test for Concavity:

$f''(x) > 0$ (positive) f(x) is concave up

$f''(x) < 0$ (negative) f(x) is concave down

#2 $f(x) = \frac{x^2+1}{x^2-4}$ Determine the concavity of the function.

$$f'(x) = \frac{(x^2-4)(2x) - (x^2+1)(2x)}{(x^2-4)^2}$$

$$f'(x) = \frac{2x^3 - 8x - 2x^3 - 2x}{(x^2-4)^2}$$

$$f'(x) = \frac{-10x}{(x^2-4)^2}$$

$$f''(x) = \frac{(x^2-4)^2(-10) - (-10x)(2)(x^2-4)^1(2x)}{\left((x^2-4)^2\right)^2}$$

$$f''(x) = \frac{-10\cancel{(x^2-4)}\left[x^2-4 - 4x^2\right]}{(x^2-4)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{-10\left[-3x^2-4\right]}{(x^2-4)^3}$$

$f''(x) = 0$

Numerator $= 0$

$-3x^2 - 4 = 0$

$x^2 = -\frac{4}{3}$

Never

$f''(x)$ undefined

Denominator $= 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

$(-\infty, -2)\ (-2, 2)\ (2, \infty)$

$x = -3$ $\quad$ $x = 0$ $\quad$ $x = 3$

sign $f''(x)$ $\quad$ $+$ $\quad$ $-$ $\quad$ $+$

concave up $(-\infty, -2)$ and $(2, \infty)$

concave down $(-2, 2)$

Points of Inflection:

If the tangent lines to the graph exist at a point where the graph changes concavity, then the point is called an inflection point. If $(c, f(c))$ is an inflection point of f, then either

$$f''(c) = 0 \quad or \quad f''(c)\ is\ undefined$$

#3 Find the points of inflection if $f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$ *

$0 = 12x^2 - 24x$

$0 = 12x(x-2)$ *

$x = 0 \qquad x = 2$

$(-\infty, 0)\ (0, 2)\ (2, \infty)$

$x = -1 \qquad x = 1 \qquad x = 3$

$+ \qquad - \qquad +$

concave up   concave down   concave up

Need y-values

$x = 0 \quad f(0) = 0^4 - 4(0)^3$

$f(0) = 0$

$x = 2 \quad f(2) = 2^4 - 4(2)^3$

$f(2) = 16 - 32$

$f(2) = -16$

Inflection points at $(0,0)$ and $(2,-16)$ due to a change in concavity.