4.4 Part 2

Tuesday, November 23, 2021 9:00 AM

AP Calculus

4.4 The Shape Of A Graph Part 2

#1 Find the inflection points for $f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

$f''(x) = 0$

$12x^2 = 0$

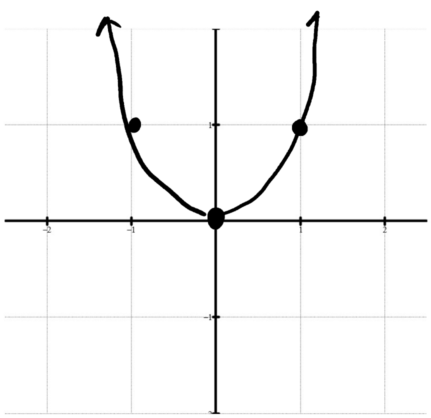
$x = 0$

$(-\infty, 0)\ (0, \infty)$

$x = -1$ $\quad$ $x = 1$

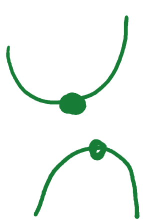
Sign of $f''(x)$: $+$ concave up; $+$ concave up

No Inflection point



2nd Derivative Test For finding max / min

1. If $f'(c) = 0$ *and* $f''(x) > 0$, c is a local minimum as f is concave up.
2. If $f'(c) = 0$ *and* $f''(x) < 0$, c is a local maximum as f is concave down.
3. If $f'(c) = 0$ *and* $f''(x) = 0$, the test fails (use the first derivative test)



#2 $f(x) = x^4 - 8x^2 + 1$ Apply the second derivative test to find any local extrema.

$f'(x) = 4x^3 - 16x$

$0 = 4x^3 - 16x$

$0 = 4x(x^2 - 4)$

$0 = 4x(x-2)(x+2)$

critical numbers

$\boxed{x = 0 \quad x = 2 \quad x = -2}$

$\boxed{f''(x) = 12x^2 - 16}$

2nd Derivative test

$x = 0$ $\quad f''(0) = 12(0)^2 - 16$

$f''(0) = -16 < 0$

concave down

$f(0) = 0 - 0 + 1$

$f(0) = 1$

Maximum $(0, 1)$

$x = 2$ $\quad f''(2) = 12(2)^2 - 16$

$f''(2) = 32 > 0$

concave up

Min $\quad f(2) = 2^4 - 8(2)^2 + 1$

$f(2) = -15$

$(2, -15)$ Minimum

$x = -2$ $\quad f''(-2) = 12(-2)^2 - 16$

$f''(-2) = 32 > 0$

concave up

$f(-2) = (-2)^4 - 8(-2)^2 + 1$

$f(-2) = -15$

$(-2, -15)$ Minimum.

#3 Find the extrema using the 2nd derivative test. Determine the concavity and any inflection points. $f(x) = -3x^5 + 5x^3$

$f'(x) = -15x^4 + 15x^2$

$0 = -15x^4 + 15x^2$

$0 = -15x^2(x^2 - 1)$

$0 = -15x^2(x-1)(x+1)$

critical numbers for $f'(x)$

$x = 0 \quad x = 1 \quad x = -1$

$f''(x) = -60x^3 + 30x$

2nd Derivative test

$x=0 \quad f''(0) = -60(0)^3 + 30(0)$ Test fails

$f''(0) = 0$

$x=1 \quad f''(1) = -60(1)^3 + 30(1)$

$f''(1) = -30 < 0$

Maximum

$f(1) = -3(1)^5 + 5(1)^3$

$f(1) = 2$

$(1, 2)$ Maximum

$x=-1 \quad f''(-1) = -60(-1)^3 + 30(-1)$

$f''(-1) = 60 - 30 = 30 > 0$

Minimum

$f(-1) = -3(-1)^5 + 5(-1)^3$

$f(-1) = 3 - 5 = -2$

$(-1, -2)$ Minimum

$(-\infty, -1)(-1, 0)(0, 1)(1, \infty)$

sign $f'(x)$ $\quad \ominus \quad \oplus \quad \oplus \quad \ominus$

$x = 0$ Not a max not a min

$f''(x) = -60x^3 + 30x$

$0 = -60x^3 + 30x$ *

$0 = -30x(2x^2 - 1)$

$0 = -30x(2x^2-1)$

$-30x = 0$ $\quad$ $2x^2-1=0$

$x = 0$ $\quad$ $x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}}$ $\quad$ $f''(x) = -30x(2x^2-1)$

Test for Concavity

	$(-\infty, -\frac{1}{\sqrt{2}})$	$(-\frac{1}{\sqrt{2}}, 0)$	$(0, \frac{1}{\sqrt{2}})$	$(\frac{1}{\sqrt{2}}, \infty)$
	$x=-1$	$x=-.5$	$x=.5$	$x=1$
sign $f''(x)$	$+$	$(+)(-)$ $-$	$(-)(-)$ $+$	$-$
	concave up	concave down	concave up	concave down

$f(-\frac{1}{\sqrt{2}}) = -3(-\frac{1}{\sqrt{2}})^5 + 5(-\frac{1}{\sqrt{2}})^3 = -1.24$

$f(0) = 0$

$f(\frac{1}{\sqrt{2}}) = -3(\frac{1}{\sqrt{2}})^5 + 5(\frac{1}{\sqrt{2}})^3 = 1.24$

3 Inflection points.