AP Calculus

4.5 Graph Sketching Part 2

Horizontal Asymptotes: If $\lim_{x\to\pm\infty} f(x) = L$ then $y = L$ is a horizontal asymptote.
A function may have two horizontal asymptotes. (section 2.7)

Slant Asymptotes: Slant asymptotes occur in a rational function (having no common factors) if the degree of the numerator is one more than the degree of the denominator. Use long division. Slant asymptote is equal to the quotient.

1. Graph the function

$$f(x) = \frac{x}{\sqrt{x^2+2}}$$

x-intercepts Numerator = 0 $X = 0$	Y-intercept Make $X = 0$ $y = \frac{0}{\sqrt{0^2+2}} = \frac{0}{\sqrt{2}} = 0$ $X = -\sqrt{X^2}$
End Behaviour Not a polynomial No vertical asymptote $\sqrt{x^2+2} = 0$ $x^2+2 = 0$ $x^2 = -2$ $X = \sqrt{X^2}$	Asymptotes $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+2}}$ $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+2}}$ $\lim_{x\to\infty} \frac{\frac{x}{x}}{\frac{\sqrt{x^2+2}}{\sqrt{x^2}}}$ $\lim_{x\to-\infty} \frac{\frac{x}{x}}{\frac{\sqrt{x^2+2}}{-\sqrt{x^2}}}$ $\lim_{x\to\infty} \frac{1}{\sqrt{\frac{x^2}{x^2}+\frac{2}{x^2}}}$ $\lim_{x\to-\infty} \frac{1}{-\sqrt{1+2/x^2}}$

$\lim_{x\to\infty} \frac{1}{\sqrt{1+2/x^2}}$ $= \frac{1}{-\sqrt{1+0}}$

$= \frac{1}{\sqrt{1+0}} = 1$ $= -1$

$y = 1$ and $y = -1$

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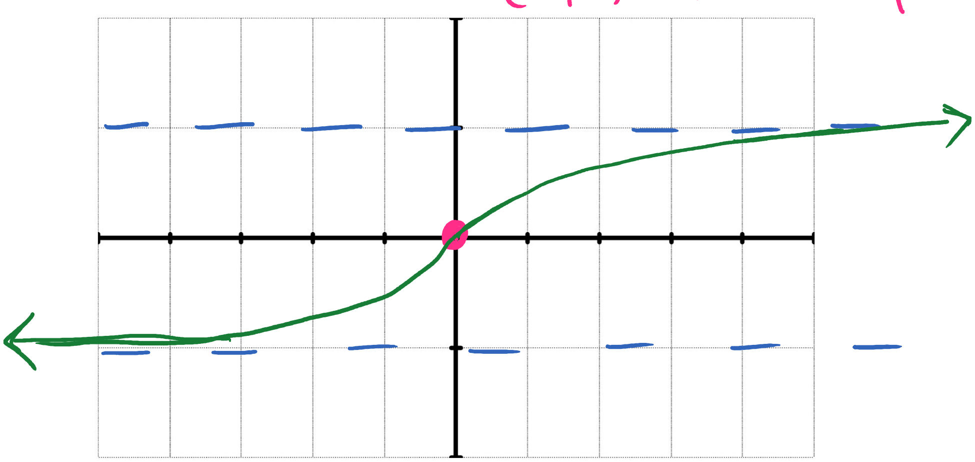
$$f(x) = \frac{x}{\sqrt{x^2+2}} = \frac{x}{(x^2+2)^{1/2}}$$

First Derivative	Sign Chart
$f'(x) = \frac{(x^2+2)^{1/2}(1) - x \cdot \frac{1}{\cancel{2}}(x^2+2)^{-1/2} \cdot \cancel{2}x}{\left((x^2+2)^{1/2}\right)^2}$ $f'(x) = \frac{(x^2+2)^{1/2} - x^2(x^2+2)^{-1/2}}{(x^2+2)}$ $f'(x) = \frac{(x^2+2)^{-1/2}\left[(x^2+2)^1 - x^2\right]}{x^2+2}$	$f'(x) = \frac{2}{(x^2+2)^{3/2}}$ $f'(x) = 0$ Numerator $= 0$ Never $f'(x)$ undefined Denominator $= 0$ $x^2+2=0$ $x^2=-2$ Never No max No min
Second Derivative	**Sign Chart**
$f''(x) = \frac{(x^2+2)^{3/2} \cdot (0) - \cancel{2} \cdot \frac{3}{\cancel{2}}(x^2+2)^{1/2} \cdot 2x}{\left((x^2+2)^{3/2}\right)^2}$ $f''(x) = \frac{-6x(x^2+2)^{1/2}}{(x^2+2)^3}$ $f''(x) = \frac{-6x}{(x^2+2)^{5/2}}$	$f''(x)=0$ $-6x=0$ $x=0$ $f''(x)$ undefined Never $(-\infty, 0)$ $(0, \infty)$ $x=-1$ $x=1$ Sign $f''(x)$: $+$ concave up; $-$ concave down $(0,0)$ Inflection point



2. Graph the function

$$f(x) = \frac{x^2 - 2x + 4}{x - 2}$$

x-intercepts	Y-intercept
$0 = x^2 - 2x + 4$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$ $x = \frac{2 \pm \sqrt{4-16}}{2}$ No x-intercepts	$x = 0$ $y = \frac{0^2 - 2(0) + 4}{0 - 2}$ $y = \frac{4}{-2} = -2$ $(0, -2)$
End Behaviour No end Behaviour Not a polynomial Vertical denominator $= 0$ $x - 2 = 0$ $x = 2$	Asymptotes Slant asymptote $x - 2 \overline{)\, x^2 - 2x + 4}$ quotient x $x^2 - 2x$ $0 + 4$ Slant asymptote $y = x$
First Derivative $f'(x) = \frac{(x-2)(2x-2) - (x^2-2x+4)(1)}{(x-2)^2}$ $f'(x) = \frac{2x^2 - 2x - 4x + 4 - x^2 + 2x - 4}{(x-2)^2}$ $f'(x) = \frac{x^2 - 4x}{(x-2)^2}$	Sign Chart $f'(x) = 0$ $x^2 - 4x = 0$ $x(x-4) = 0$ $x = 0 \quad x = 4$ $f'(x)$ undefined $x - 2 = 0$ $x = 2$ $(-\infty, 0)\ (0, 2)\ (2, 4)\ (4, \infty)$ $x = -1$ (+) $\quad x = 1$ (−) $\quad x = 3$ (−) $\quad x = 5$ (+) $f(0) = 0$ $(0, -2)$ Max $f(4) = \frac{4^2 - 2(4) + 4}{4 - 2}$ $f(4) = \frac{16 - 8 + 4}{2}$ $f(4) = 6$ $(4, 6)$ Min

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$$f'(x) = \frac{x^2-4x}{(x-2)^2}$$

Second Derivative	Sign Chart
$f''(x) = \frac{(x-2)^2(2x-4) - (x^2-4x)(2)(x-2)^1(1)}{((x-2)^2)^2}$ $f''(x) = \frac{\cancel{(x-2)}[(x-2)(2x-4) - (x^2-4x)(2)]}{(x-2)^{\cancel{4}\,3}}$ $f''(x) = \frac{2x^2-4x-4x+8-2x^2+8x}{(x-2)^3}$ $f''(x) = \frac{8}{(x-2)^3}$	$f''(x)=0$ Never $f''(x)$ undefined $x-2=0$ $x=2$ $(-\infty,2)\ (2,\infty)$ $x=0$ $\quad$ $x=3$ $\ominus$ $\quad$ $\oplus$ concave down $\quad$ concave up

No inflection point

$x=2$ Asymptote