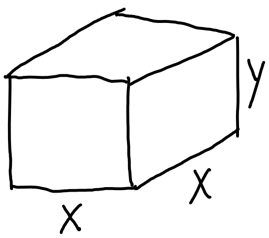
AP Calculus

4.6 Optimization Problems

1. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce the maximum volume?



$S = 108$

$108 = x^2 + 4xy$

$108 - x^2 = 4xy$

$\frac{108 - x^2}{4x} = y$

$V = x^2 y$

$V = x^2\left[\frac{108 - x^2}{4x}\right]$

$V = \frac{108x^2 - x^4}{4x}$

$V = 27x - \frac{1}{4}x^3$

$0 \le x \le \sqrt{108}$

$V' = 27 - \frac{1}{4}(3x^2)$

$V' = 27 - \frac{3}{4}x^2$

$0 = 27 - \frac{3}{4}x^2$

$\frac{3}{4}x^2 = 27$

$x^2 = 27\left(\frac{4}{3}\right)$

$x^2 = 36$

$x = 6$

$(0, 6)\ (6, \sqrt{108})$

$x=1$ $x=10$

sign V' $\oplus$ $\ominus$

$x = 6$ Max

find y when $x = 6$

$y = \frac{108 - 6^2}{4(6)}$

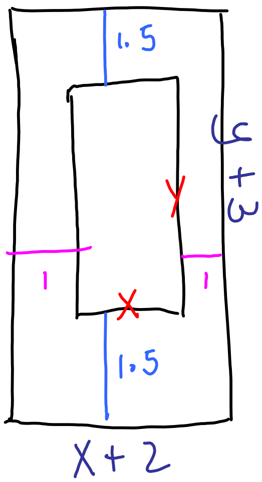
$y = \frac{108 - 36}{24}$

$y = \frac{72}{24} = 3$

Dimensions
6 by 6 by 3 inches.

- Identify the quantity to be maximized / minimized
- Write an equation in one variable (use a secondary equation)
- Determine domain
- Use calculus to find max / min

2. A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are each 1.5 inches. The margins on each side are 1 inch. What should the dimensions of the page be so that the least amount of paper is used?



$xy = 24$

$x = \frac{24}{y}$

$A_{page} = (x+2)(y+3)$

$A = \left(\frac{24}{y}+2\right)(y+3)$

$A = 24 + \frac{72}{y} + 2y + 6$

$A = 30 + \frac{72}{y} + 2y$

$y > 0$

$A' = 0 + 72(-1)y^{-2} + 2$

$A' = -\frac{72}{y^2} + 2$

$A' = \frac{-72+2y^2}{y^2}$

$-72 + 2y^2 = 0$

$2y^2 = 72$

$y^2 = 36$

$y = 6$

$(0,6)\ (6,\infty)$

$y=1 \quad y=10$

sign A' $\quad - \quad +$

$y = 6$ minimum

find x

$x = \frac{24}{6}$

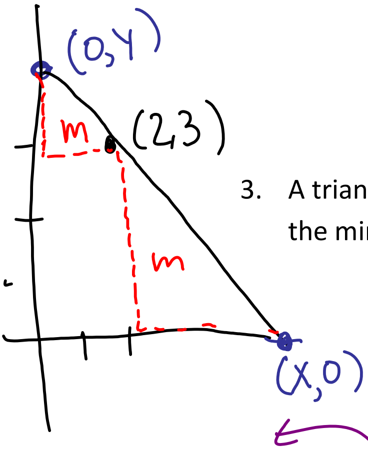
$x = 4$

dimensions of page

$(x+2)$ by $(y+3)$

$x=4 \qquad y=6$

6 by 9 inches



3. A triangle is formed in the first quadrant using the axes and a line through the point (2,3). Find the minimum area of the triangle.

$A = \frac{XY}{2}$

$\frac{Y-3}{0-2} = \frac{0-3}{X-2}$

$X > 2$

$\frac{Y-3}{-2} = \frac{-3}{X-2}$

$Y-3 = \frac{6}{X-2}$

$Y = \frac{6}{X-2} + 3$

$A = \frac{X}{2}\left[\frac{6}{X-2} + 3\right]$

$A = \frac{X}{2}\left[\frac{6+3X-6}{X-2}\right]$

$A = \frac{3X^2}{2(X-2)}$

$A' = \frac{2(X-2)(6X) - 3X^2(2)}{4(X-2)^2}$

$= \frac{6X^2 - 12X - 3X^2}{2(X-2)^2}$

$= \frac{3X^2 - 12X}{2(X-2)^2}$

$3X^2 - 12X = 0$

$3X(X-4) = 0$

$\cancel{X=0}$ $X = 4$

$(2,4)\ (4,\infty)$

$X=3 \quad X=5$

sign A' $\ominus$ $\oplus$

$X = 4$ minimum

$X = 4$

$Y = \frac{6}{4-2} + 3$

$Y = 6$

$\text{Area} = \frac{(4)(6)}{2}$

$= 12 \text{ units}^2$

4. Find the points on the parabola, $y = 6 - x^2$ that are closest to the point (0,3)

5. Four feet of wire is to be used to form a square and a circle. How much of the wire should be used for the square and how much should be used for the circle to enclose the minimum total area?