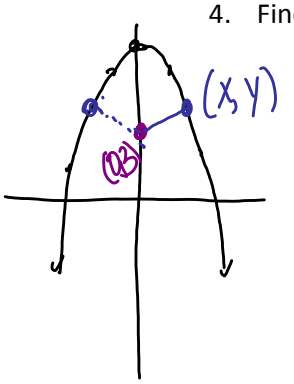
3. A triangle is formed in the first quadrant using the axes and a line through the point (2,3). Find the minimum area of the triangle.

4. Find the points on the parabola, $y = 6 - x^2$ that are closest to the point (0,3)



$d = \sqrt{(x-0)^2 + (y-3)^2}$

$y = 6 - x^2$

$d = \sqrt{x^2 + (y-3)^2}$

$d = \sqrt{x^2 + (6-x^2-3)^2}$

$d = \sqrt{x^2 + (3-x^2)^2}$

$d = \sqrt{x^2 + 9 - 6x^2 + x^4}$

$d = \sqrt{x^4 - 5x^2 + 9}$

$d' = \frac{1}{2}(x^4 - 5x^2 + 9)^{-\frac{1}{2}} \cdot (4x^3 - 10x)$

$d' = \frac{4x^3 - 10x}{2(x^4 - 5x^2 + 9)^{1/2}}$

when d' is undefined

when $d' = 0$

$4x^3 - 10x = 0$

$2x(2x^2 - 5) = 0$

$x = 0 \quad x = \pm\sqrt{5/2}$

$(-\infty, -\sqrt{5/2}) \quad (-\sqrt{5/2}, 0) \quad (0, \sqrt{5/2}) \quad (\sqrt{5/2}, \infty)$

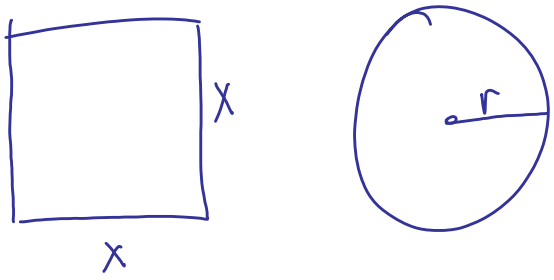
$x = -2 \quad x = -1 \quad x = 1 \quad x = 2$

sign d' $\ominus$ $\oplus$ $\ominus$ $\oplus$

$x = \pm\sqrt{5/2}$ Min ~~$x = 0$ Max~~

$y = 6 - \left(\sqrt{\frac{5}{2}}\right)^2 = 6 - \frac{5}{2}$ $\quad y = \frac{7}{2}$ $\quad \left(\sqrt{\frac{5}{2}}, \frac{7}{2}\right) \left(-\sqrt{\frac{5}{2}}, \frac{7}{2}\right)$

5. Four feet of wire is to be used to form a square and a circle. How much of the wire should be used for the square and how much should be used for the circle to enclose the minimum total area?



$A = x^2 + \pi r^2$

$4 = 4x + 2\pi r \qquad 0 \le x \le 1$

$4 - 4x = 2\pi r$

$\frac{4-4x}{2\pi} = r$

$\frac{2-2x}{\pi} = r$

$A = x^2 + \pi\left[\frac{2-2x}{\pi}\right]^2$

$A = x^2 + \frac{\pi(4-8x+4x^2)}{\pi^2}$

$A = x^2 + \frac{4-8x+4x^2}{\pi}$

$A' = 2x + 0 - \frac{8}{\pi} + \frac{8x}{\pi}$

$0 = 2x - \frac{8}{\pi} + \frac{8x}{\pi}$

$\frac{8}{\pi} = 2x + \frac{8x}{\pi}$

$\frac{8}{\pi} = x\left(2 + \frac{8}{\pi}\right)$

$\frac{\frac{8}{\pi}}{2+\frac{8}{\pi}} = x \qquad x = \frac{4}{\pi+4}$

$\left(0, \frac{4}{\pi+4}\right) \quad \left(\frac{4}{\pi+4}, 1\right)$

$x = .5 \qquad x = .7$

Sign A': $-$ then $+$

$x = \frac{4}{\pi+4}$ Min

square $= 4\left(\frac{4}{\pi+4}\right) = \frac{16}{\pi+4}$

$x = 0$

$A = \frac{4}{\pi} \approx 1.27$

$x = 1$

$A = 1$

$x = \frac{4}{\pi+4}$

$A = \left(\frac{4}{\pi+4}\right)^2 + \frac{4 - 8\left(\frac{4}{\pi+4}\right) + 4\left(\frac{4}{\pi+4}\right)^2}{\pi}$

$A = .56$

circle $= 4 - \frac{16}{\pi+4}$

$= \frac{4(\pi+4) - 16}{\pi+4}$

$= \frac{4\pi + 16 - 16}{\pi+4}$

$= \frac{4\pi}{\pi+4}$