4.8 Antiderivatives

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Antiderivatives: A function $F(x)$ is an antiderivative of $f(x)$ if

$F'(x) = f(x)$ for all x on (a, b)

$F(x) = \frac{1}{4}x^4$ is an antiderivative for $f(x) = x^3$

$F'(x) = \frac{1}{4}(4)x^3$

$F'(x) = x^3$

Antiderivatives are NOT unique

$$\left.\begin{aligned} F(x) &= \tfrac{1}{4}x^4 \\ F(x) &= \tfrac{1}{4}x^4 + 2 \\ F(x) &= \tfrac{1}{4}x^4 - 11 \end{aligned}\right\} \text{all antiderivatives for } f(x) = x^3$$

General Antiderivative: Let $F(x)$ be an antiderivative of $f(x)$ on (a, b). Then every other antiderivative is of the form $F(x) + c$ where c is a constant.

The process of finding an antiderivative is called integration.

$$\int f(x)\,dx = F(x) + C$$

Notation for finding antiderivatives:

Given y' find y

Given $\frac{dy}{dx}$ find y

1. If $f(x) = 4x^3 + 6x^2 + 11$ find the general antiderivative.

$$F(x) = \int 4x^3 + 6x^2 + 11 \; dx$$

$$F(x) = \frac{4x^4}{4} + \frac{6x^3}{3} + \frac{11x^1}{1} + C$$

$$F(x) = x^4 + 2x^3 + 11x + C$$

2. If $f(x) = 3x^{\frac{-2}{3}}$ find the general antiderivative.

$$F(x) = \int 3x^{-2/3} \, dx$$

$$F(x) = \frac{3x^{\frac{1}{3}}}{1/3} + C$$

$$F(x) = 9x^{1/3} + C$$

3. $\frac{dy}{dx} = 2\sin x$ find y

$$y = \int 2\sin x \, dx$$

$$y = 2(-\cos x) + c$$

$$y = -2\cos x + c$$

4. $\int \frac{x^2+2x-3}{x^4} dx$

$$= \int \frac{x^2}{x^4} + \frac{2x}{x^4} - \frac{3}{x^4} \, dx$$

$$= \int x^{-2} + 2x^{-3} - 3x^{-4} \, dx$$

$$= \frac{x^{-1}}{-1} + \frac{2x^{-2}}{-2} - \frac{3x^{-3}}{-3} + C$$

$$= \frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$$

5. Given $F(x) = \int (3x^2 - 1)\, dx$ and $F(2) = 4$ find $F(x)$

$$F(x) = \int (3x^2 - 1)\, dx$$

$$F(x) = \frac{3x^3}{3} - \frac{x^1}{1} + C$$

$$F(x) = x^3 - x + C$$

$$F(2) = 4$$

$$4 = (2)^3 - (2) + C$$

$$4 = 8 - 2 + C$$

$$4 = 6 + C$$

$$-2 = C$$

$$F(x) = x^3 - x - 2$$

6. $F'(x) = \frac{1}{x^2}$ $x > 0$ Find the solution if $F(1) = 0$

$$F(x) = \int \frac{1}{x^2}\,dx$$

$$F(x) = \int x^{-2}\,dx$$

$$F(x) = \frac{x^{-1}}{-1} + C$$

$$F(x) = -\frac{1}{x} + C$$

General Solution

$$F(1) = 0$$

$$0 = \frac{-1}{1} + C$$

$$0 = -1 + C$$

$$1 = C$$

$$F(x) = -\frac{1}{x} + 1$$

Particular solution