5.1 Part 1

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AP Calculus

5.1 Part 1 Sigma Notation and Approximating Area

Sigma notation: the sum of n terms $a_1, a_2, a_3, a_4, \ldots a_{n-1}, a_n$

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots . a_n$$

#1 Evaluate

$$\sum_{i=1}^{6} i = 1 + 2 + 3 + 4 + 5 + 6$$

$$\sum_{i=3}^{7} i^2 = 3^2 + 4^2 + 5^2 + 6^2 + 7^2$$

$$\sum_{i=1}^{n} \frac{1}{n}(i^2+1) = \frac{1}{n}\left[(1^2+1) + (2^2+1) + (3^2+1) + \ldots (n^2+1)\right]$$

$$\sum_{i=1}^{n} f(x_i)\Delta x = f(x_1)\Delta x + f(x_2)\Delta x + \ldots \ldots f(x_n)\Delta x$$

Properties of Summations:

$$\sum_{i=1}^{n} k a_i = K \sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

Sigma Formulas:

$$\sum_{i=1}^{n} c = cn$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

#2 Evaluate

$$\sum_{i=1}^{n} \frac{i+1}{n^2} \quad for\ n = 10, 100, 1000, 10000$$

$$= \frac{1}{n^2} \sum_{i=1}^{n} i+1$$

$$= \frac{1}{n^2}\left[\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1\right]$$

$$= \frac{1}{n^2}\left[\frac{n(n+1)}{2} + 1(n)\right]$$

$$= \frac{\cancel{n}(n+1)}{2n^{\cancel{2}}} + \frac{\cancel{n}}{n^{\cancel{2}}}$$

$$= \frac{n+1}{2n} + \frac{1\cdot 2}{n\cdot 2}$$

$$= \frac{n+3}{2n}$$

$$y = \frac{n+3}{2n}$$

$n=10$ $y = 0.65$

$n=100$ $y = 0.515$

$n=1000$ $y = 0.5015$

$n=10000$ $y = 0.50015$

$$\lim_{n\to\infty} \frac{n+3}{2n}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}}{2\cancel{n}} + \frac{3}{2n}$$

$$= \frac{1}{2} + 0$$

$$= 1/2$$

#3 Find the sum

$$\sum_{k=3}^{5} (k^2 - 2k - 3) = \sum_{K=1}^{5} (K^2 - 2K - 3) - \sum_{K=1}^{2} (K^2 - 2K - 3)$$

$$\sum_{k=3}^{5}(k^2-2k-3) = \sum_{K=1}^{5}(k^2-2k-3) - \sum_{K=1}^{2}(k^2-2k-3)$$

$$= \sum_{K=1}^{5}K^2 - 2\sum_{K=1}^{5}K - \sum_{K=1}^{5}3 - \sum_{K=1}^{2}K^2 + 2\sum_{K=1}^{2}K + \sum_{K=1}^{2}3$$

$$= \frac{5\cancel{(5+1)}(10+1)}{\cancel{6}} - \cancel{2}\frac{(5)(5+1)}{\cancel{2}} - 3(5) - \frac{2\cancel{(2+1)}(4+1)}{\cancel{6}} + 2\frac{\cancel{(2)}(2+1)}{\cancel{2}} + 3(2)$$

$$= 55 - 30 - 15 - 5 + 6 + 6$$

$$= 17$$

#4 Evaluate

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n^3}(i-1)^2$$

$$= \lim_{n\to\infty}\frac{1}{n^3}\sum_{i=1}^{n}(i-1)^2$$

$$= \lim_{n\to\infty}\frac{1}{n^3}\sum_{i=1}^{n}(i^2-2i+1)$$

$$= \lim_{n\to\infty}\frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6} - \frac{\cancel{2}(n)(n+1)}{\cancel{2}} + 1(n)\right]$$

$$= \lim_{n\to\infty}\left[\frac{\cancel{n}(2n^2+3n+1)}{6n^{\cancel{3}\,2}} - \frac{\cancel{n}(n+1)}{n^{\cancel{3}\,2}} + \frac{\cancel{n}}{n^{\cancel{3}\,2}}\right]$$

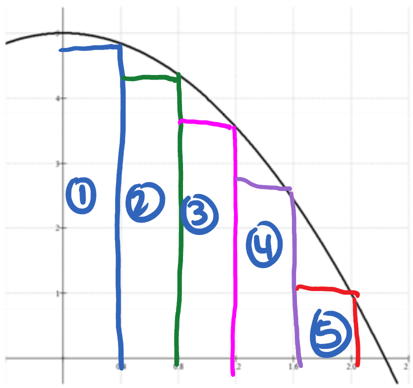
$$= \lim_{n\to\infty}\left(\frac{2\cancel{n^2}}{6\cancel{n^2}} + \frac{3\cancel{n}}{6n^{\cancel{2}}} + \frac{1}{6n^2} - \frac{\cancel{n}}{n^{\cancel{2}}} - \frac{1}{n^2} + \frac{1}{n^2}\right)$$

$$= \frac{2}{6}$$

$$= 1/3$$

5. Using the function $f(x) = -x^2 + 5$ estimate the area under the curve from $x=0$ to $x=2$ by calculating the area of five rectangles of equal width and finding the sum.

a) Using inscribed rectangles



$A_1 = f\left(\frac{2}{5}\cdot 1\right)\cdot \frac{2}{5} = 1.936$

$A_2 = f\left(\frac{2}{5}\cdot 2\right)\cdot \frac{2}{5} = 1.744$

$A_3 = f\left(\frac{2}{5}\cdot 3\right)\cdot \frac{2}{5} = 1.424$

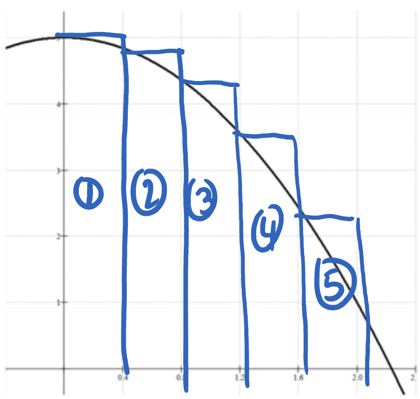
$A_4 = f\left(\frac{2}{5}\cdot 4\right)\cdot \frac{2}{5} = 0.976$

$A_5 = f\left(\frac{2}{5}\cdot 5\right)\cdot 2/5 = 0.4$

6.48

$\text{width} = \Delta x = \frac{2-0}{5} = \frac{2}{5}$

b) Using circumscribed rectangles



$A_1 = f\left(\frac{2}{5}\cdot 0\right)\cdot \frac{2}{5} = 2$

$A_2 = f\left(\frac{2}{5}\cdot 1\right)\cdot \frac{2}{5} = 1.936$

$A_3 = f\left(\frac{2}{5}\cdot 2\right)\cdot \frac{2}{5} = 1.744$

$A_4 = f\left(\frac{2}{5}\cdot 3\right)\cdot 2/5 = 1.424$

$A_5 = f\left(2/5\cdot 4\right)\cdot 2/5 = 0.976$

8.08

$6.48 < \text{Actual Area} < 8.08$

A better estimation for the area under the curve could be obtained by:

More rectangle -> narrow rectangles

Trapezoid

Midpoint of Rectangles