5.1 Part 2

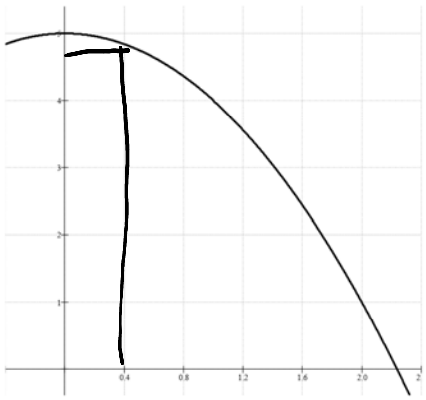
Wednesday, December 11, 2019 1:42 PM

AP Calculus

5.1 Approximating and Computing Area

1. Using the function $f(x) = -x^2 + 5$ estimate the area under the curve from $x=0$ to $x=2$ using 5 rectangles and the summation formulas.

a) Using inscribed rectangles R_5



width $= \frac{2-0}{5} = \frac{2}{5}$

height (right side)

right endpoint $\frac{2}{5}i$

height $f\left(\frac{2}{5}i\right)$

Area = (width) height

$$\text{Area} = \sum_{i=1}^{5} \left(\frac{2}{5}\right) \cdot f\left(\frac{2}{5}i\right)$$

$$A = \frac{2}{5} \sum_{i=1}^{5} -\left(\frac{2}{5}i\right)^2 + 5$$

$$A = \frac{2}{5} \sum_{i=1}^{5} -\frac{4}{25}i^2 + 5$$

$$A = \frac{2}{5}\left[\frac{-4\,\cancel{(5)}\cancel{(5+1)}(10+1)}{\cancel{25}\,5 \cdot \cancel{6}} + 5(5)\right]$$

$$A = \frac{2}{5}\left[-\frac{44}{5} + 25\right]$$

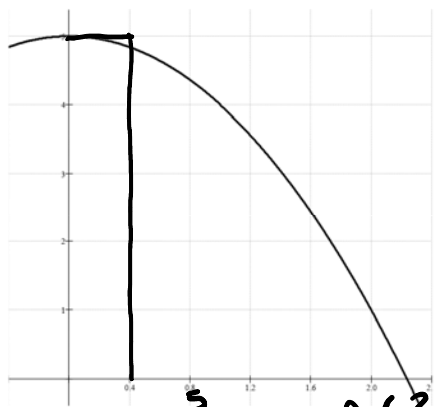
$$A = 6.48$$

$$\text{width} = \frac{2-0}{5} = \frac{2}{5}$$

b) Using circumscribed rectangles L_5

$$\text{left endpoint} = \frac{2}{5}(i-1)$$

$$\text{height} = f\left(\frac{2}{5}(i-1)\right)$$



$$\text{Area} = \sum_{i=1}^{5} \frac{2}{5} \cdot f\left(\frac{2}{5}(i-1)\right)$$

$$A = \frac{2}{5} \sum_{i=1}^{5} -\left(\frac{2}{5}(i-1)\right)^2 + 5$$

$$A = \frac{2}{5} \sum_{i=1}^{5} -\frac{4}{25}(i^2 - 2i + 1) + 5$$

$$A = \frac{2}{5}\left[\frac{-4}{25}\left(\frac{5(5+1)(10+1)}{6} - 2\frac{(5)(5+1)}{2} + 1(5)\right) + 5(5)\right]$$

$$A = \frac{2}{5}\left[\frac{-44}{5} + \frac{24}{5} - \frac{4}{5} + 25\right]$$

$$A = 8.08$$

If f(x) is continuous and non-negative on [a,b], then the endpoint and midpoint approximations approach one and the same limit as $N \to \infty$.

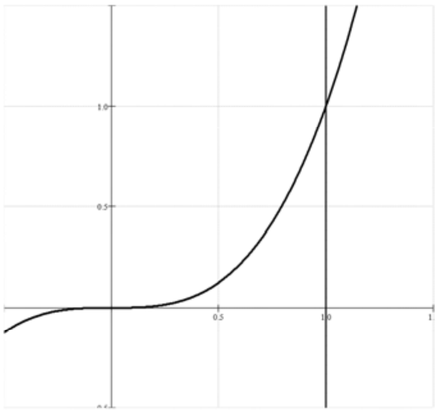
$$\lim_{N\to\infty} R_N = \lim_{N\to\infty} L_N = \lim_{N\to\infty} M_N$$

$$\Delta x = \text{width} = \frac{b-a}{n}$$

$$Area = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$

c_i = point on interval (left, right or midpoint)

2. Find the area of the region bounded by the graph of $f(x) = x^3$, the x-axis and the vertical lines x=0 and x=1.



$\Delta x = \frac{1-0}{n} = \frac{1}{n}$

C_i= any value in the ith subinterval

right

$c_i = \frac{1}{n} i = \frac{i}{n}$

$$Area = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$

$$Area = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\cdot \frac{1}{n}$$

$$A = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3$$

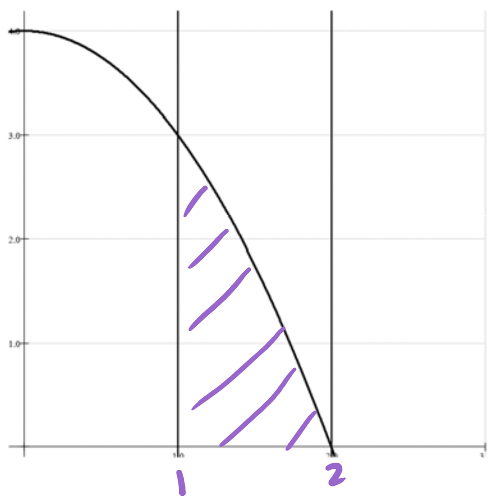
$$A = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \frac{i^3}{n^3}$$

$$A = \lim_{n\to\infty} \frac{1}{n^{\cancel{4}\,2}} \left[\frac{\cancel{n^2}(n+1)^2}{4}\right]$$

$$A = \lim_{n\to\infty} \frac{n^2+2n+1}{4n^2}$$

$$A = \lim_{n\to\infty} \frac{1 + 2/n + 1/n^2}{4} = \frac{1}{4}$$

3. Find the area bounded by $f(x) = 4 - x^2$, the x-axis, and the vertical lines x=1 and x=2



$$\text{width} = \frac{2-1}{n} = \frac{1}{n}$$

$$\text{right} = 1 + \frac{1}{n}i$$

$$\text{right} = a + \Delta x i$$

$$\text{height} = f\left(1 + \frac{i}{n}\right)$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(1+\frac{i}{n}\right)\cdot\frac{1}{n}$$

$$A = \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} 4 - \left(1+\frac{i}{n}\right)^2$$

$$A = \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \left(4 - 1 - \frac{2i}{n} - \frac{i^2}{n^2}\right)$$

$$A = \lim_{n\to\infty} \frac{1}{n}\left[3(n) - \frac{\cancel{2}}{\cancel{n}}\frac{(\cancel{n})(n+1)}{\cancel{2}} - \frac{1}{n^{\cancel{2}}}\frac{(\cancel{n})(n+1)(2n+1)}{6}\right]$$

$$A = \lim_{n\to\infty} \frac{1}{n}\left[3n - n - 1 - \frac{2n^2+3n+1}{6n}\right]$$

$$A = \lim_{n\to\infty} \left[2 - \frac{1}{n} - \frac{2n^2+3n+1}{6n^2}\right]$$

$$A = \lim_{n\to\infty} \left[2 - \frac{1}{n} - \frac{2\cancel{n^2}}{6\cancel{n^2}} - \frac{3\cancel{n}}{6n^{\cancel{2}}} - \frac{1}{6n^2}\right]$$

$$A = 2 - \frac{1}{3} = \frac{5}{3}$$