AP Calculus

5.2 The Definite Integral

as $n \to \infty$
width of rectangles get smaller $\therefore$ $\Delta x \to 0$

If f is defined on the closed interval [a,b] and the limit

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(c_i)\Delta x$$

exists then f is integrable on [a,b] and the limit is denoted by

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(c_i)\Delta x = \int_a^b f(x)dx$$

This is a definite integral

Definite integral is a number
Indefinite integral is a family of functions

1. Evaluate using summation formulas.

$$\int_{-1}^{2} 2x\,dx$$

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$

$f(x) = 2x$

c_i = right endpoint
$c_i = a + \Delta x i$
$c_i = -1 + \frac{3}{n} i$

$= \lim_{n \to \infty} \sum_{i=1}^{n} f\left(-1 + \frac{3i}{n}\right) \cdot \frac{3}{n}$

$= \lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^{n} 2\left(-1 + \frac{3i}{n}\right)$

$= \lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^{n} \left(-2 + \frac{6i}{n}\right)$

$= \lim_{n \to \infty} \frac{3}{n}\left[-2(n) + \frac{\cancel{6}^{3}}{\cancel{n}} \frac{(\cancel{n})(n+1)}{\cancel{2}}\right]$

$= \lim_{n \to \infty} \frac{3}{n}\left[-2n + 3n + 3\right]$

$= \lim_{n \to \infty} \left(\frac{3\cancel{n}}{\cancel{n}} + \frac{9}{n}\right)$

$= 3$

Definite Integral as an Area of a Region

$$\int_a^b f(x)dx = $$ Signed area of a region between the graph and the x-axis over $[a,b]$

2. Evaluate using geometry

$$\int_{-1}^{2} 2x dx$$

$f(x)=2x$

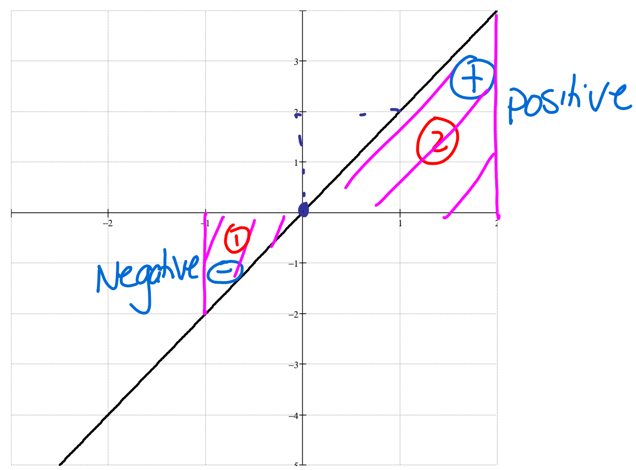
interval $[-1,2]$

$$\text{Area} = A_1 + A_2$$

$$= \frac{(1)(-2)}{2} + \frac{(2)(4)}{2}$$

$$= -1 + 4$$

$$= 3$$



3. Evaluate using geometry

$$\int_0^2 |2x-1| dx$$

① $y=2x-1$

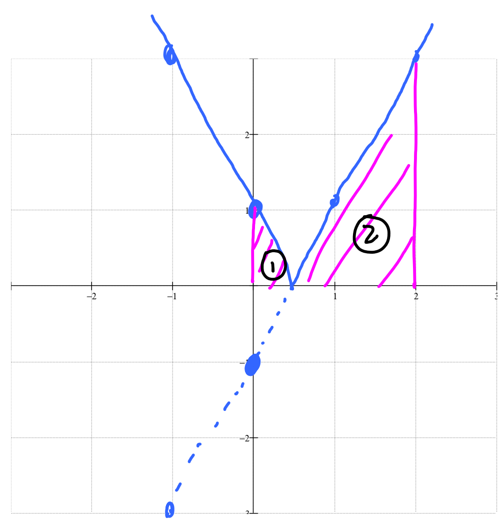
② $y=|2x-1|$

x-intercept of $y=2x-1$

$$0=2x-1$$

$$1=2x$$

$$\frac{1}{2}=x$$



$$= A_1 + A_2$$

$$= \frac{(\frac{1}{2})(1)}{2} + \frac{(\frac{3}{2})(3)}{2}$$

$$= \frac{1}{2}\cdot\frac{1}{2} + \frac{9}{2}\cdot\frac{1}{2}$$

$$= \frac{1}{4} + \frac{9}{4}$$

$$= 10/4$$

$$= 5/2$$

Properties of definite integrals

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \qquad a < c < b$$

$$\int_a^b kf(x)dx = K\int_a^b f(x)dx$$

$$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b f(x)dx \geq 0 \quad \text{if } f(x) \text{ is non-negative on } [a,b]$$

$$\int_a^b f(x)dx \geq \int_a^b g(x)dx \quad \text{if } f(x) \geq g(x) \text{ on } [a,b]$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

4. Evaluate

$$\int_1^3 (-x^2 + 4x - 3)dx \qquad \textit{given that} \quad \int_1^3 x^2dx = \frac{26}{3} \quad \int_1^3 xdx = 4 \quad \int_1^3 dx = 2$$

$$= \int_1^3 -x^2\,dx + \int_1^3 4x\,dx + \int_1^3 -3\,dx$$

$$= -\int_1^3 x^2\,dx + 4\int_1^3 x\,dx - 3\int_1^3 dx$$

$$= -\frac{26}{3} + 4(4) - 3(2)$$

$$= -\frac{26}{3} + 16 - 6$$

$$= -\frac{26}{3} + \frac{48}{3} - \frac{18}{3} = \frac{4}{3}$$

5. Evaluate the following given that

$$\int_0^9 f(x)dx = 6.75 \qquad \boxed{\int_2^0 f(x)dx = -5} \Rightarrow \int_0^2 f(x)dx = 5$$

$a) \int_2^9 f(x)dx$

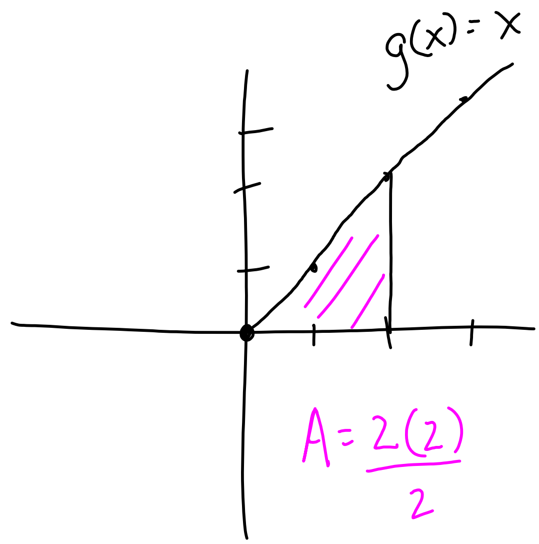
$$= \int_0^9 f(x)dx - \int_0^2 f(x)dx$$

$$= 6.75 - 5$$

$$= 1.75$$

$b) \int_0^2 (f(x) + x)dx$

$$= \int_0^2 f(x)dx + \boxed{\int_0^2 x\,dx} \rightarrow$$



$$A = \frac{2(2)}{2}$$

$$A = 2$$

$$= 5 + 2$$

$$= 7$$