5.3 Fundamental Theorem of Calculus Part 1

Note Title 1/13/2015

Assume that $f(x)$ is continuous on $[a,b]$
If $F(x)$ is an antiderivative of $f(x)$ on $[a,b]$
then

$$\int_a^b f(x) = F(b) - F(a)$$

First Fundamental Theorem of Calculus

#1 Calculate $\int_{-1}^{2} 2x\,dx$

if $f(x) = 2x$ then the antiderivative is

$$F(x) = \frac{2x^2}{2} = x^2 + C$$

$$\int_{-1}^{2} 2x\,dx = F(2) - F(-1)$$

$$= (2)^2 + c - ((-1)^2 + c)$$

$$= 4 + c - 1 - c$$

$$= 3$$

#2 $\int_0^1 x^3\,dx$ $\quad f(x) = x^3 \quad F(x) = \frac{x^4}{4} + C$

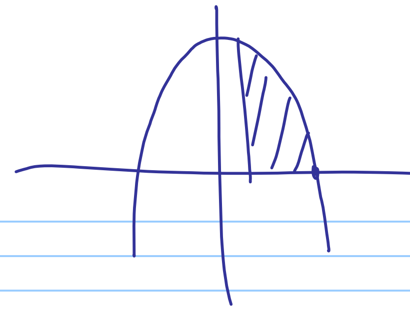
$$= \frac{1}{4}x^4 \Big|_0^1$$

$$= \frac{1}{4}(1)^4 - \frac{1}{4}(0)^4$$

$$= \frac{1}{4} - 0$$

$$= \frac{1}{4}$$

#3 $\int_1^2 (4-x^2)\,dx$



$= 4x - \frac{x^3}{3} \Big|_1^2$

$= \left(4(2) - \frac{(2)^3}{3}\right) - \left(4(1) - \frac{(1)^3}{3}\right)$

$= 8 - \frac{8}{3} - 4 + \frac{1}{3}$

$= \frac{24}{3} - \frac{8}{3} - \frac{12}{3} + \frac{1}{3}$

$= \frac{5}{3}$

#4 $\int_0^{\pi/4} \sec^2 x\,dx$

$\int \sec^2 x\,dx = \tan x + c$

$= \tan x \Big|_0^{\pi/4}$

$= \tan\frac{\pi}{4} - \tan 0$

$= 1 - 0$

$= 1$

#5 $\int_0^8 \sqrt[3]{x}\,dx$

$\int_0^8 x^{\frac{1}{3}}\,dx$

$\frac{1}{3} + 1$

$\frac{1}{3} + \frac{3}{3}$

$\frac{4}{3}$

$= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} \Big|_0^8$

$$= \frac{3}{4} x^{4/3} \Big|_0^8$$

$$= \frac{3}{4}\left(\sqrt[3]{8}\right)^4 - \frac{3}{4}(0)^{4/3}$$

$$= \frac{3}{4}(2)^4 - 0$$

$$= \frac{3}{4}(16)$$

$$= 12$$

#6 $\displaystyle\int_0^2 |2x-1|\,dx$

x intercept

$2x-1=0$

$2x=1$

$x=\frac{1}{2}$

$$|2x-1| = \begin{cases} -(2x-1) & x<\frac{1}{2} \\ 2x-1 & x \geq \frac{1}{2} \end{cases}$$

$$= \int_0^{\frac{1}{2}} -(2x-1)\,dx + \int_{\frac{1}{2}}^2 (2x-1)\,dx$$

$$= -\left[x^2 - x\right]_0^{\frac{1}{2}} + \left[x^2 - x\right]_{1/2}^2$$

$$= -\left[\left(\frac{1}{2}\right)^2 - \frac{1}{2} - 0\right] + \left[2^2 - 2 - \left(\left(\frac{1}{2}\right)^2 - \frac{1}{2}\right)\right]$$

$$= -\left[\frac{1}{4} - \frac{1}{2}\right] + \left[4 - 2 - \frac{1}{4} + \frac{1}{2}\right]$$

$$= -\frac{1}{4} + \frac{1}{2} + 4 - 2 - \frac{1}{4} + \frac{1}{2}$$

$$= 5/2$$

#7 Find the area bounded by $y = 2x^2 - 3x + 2$, the x-axis, and the lines $x=0$ and $x=2$.

$$= \int_0^2 (2x^2 - 3x + 2)\,dx$$

$$= \left(\frac{2x^3}{3} - \frac{3x^2}{2} + 2x\right)\Big|_0^2$$

$$= \left(\frac{2}{3}(2)^3 - \frac{3}{2}(2)^2 + 2(2)\right) - (0)$$

$$= \frac{16}{3} - 6 + 4$$

$$= \frac{16}{3} - \frac{18}{3} + \frac{12}{3}$$

$$= 10/3$$