5.4: Trigonometric Equations Part I

Ex. #1: Solve using exact values $4sin\left(\boxed{x - \frac{\pi}{3}}\right) = 2 \qquad 0 \leq x < 2\pi$

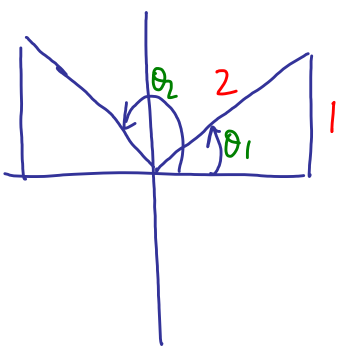
$\theta = x - \frac{\pi}{3}$

$4\sin\theta = 2$

$\sin\theta = \frac{2}{4}$

$\sin\theta = \frac{1}{2}$

1. Subsitute θ for the angle/phase shift
2. Isolate trig function
3. Solve for θ (special Δ's)
4. Sub back to solve for x.



The two angles have the same reference angle

Ref $\angle = \frac{\pi}{6}$

$\theta_1 = \frac{\pi}{6}$

$\theta_2 = \pi - \frac{\pi}{6}$

$= \frac{6\pi}{6} - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

$\theta = x - \frac{\pi}{3}$

$x_1 - \frac{\pi}{3} = \frac{\pi}{6}$

$x_2 - \frac{\pi}{3} = \frac{5\pi}{6}$

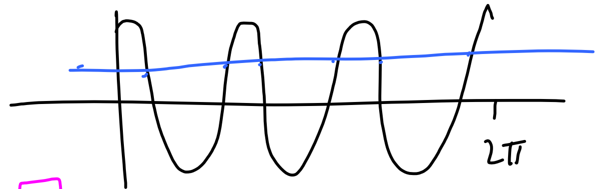
$x_1 - \frac{2\pi}{6} = \frac{\pi}{6}$

$x_2 - \frac{2\pi}{6} = \frac{5\pi}{6}$

$x_1 = \frac{3\pi}{6}$

$x_2 = \frac{7\pi}{6}$

$x_1 = \pi/2$



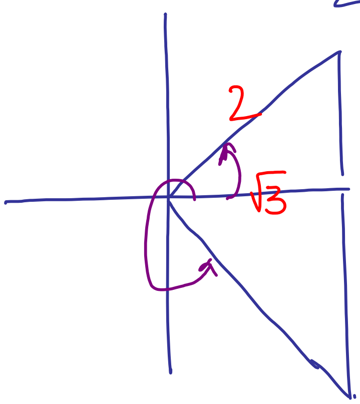
Ex. #2: Solve using exact values. $2\cos\boxed{3x} = \sqrt{3}$ $0 \le x < 2\pi$

$\theta = 3x$

$2\cos\theta = \sqrt{3}$

$\cos\theta = \frac{\sqrt{3}}{2}$

1. Substitution for the angle/period change
2. Isolate the trig function
3. Solve for θ (special Δ's)
4. Sub back (solve for x)
5. Add on period to find the other solutions



ref $\angle = \frac{\pi}{6}$

$\theta_1 = \frac{\pi}{6}$

$\theta_2 = 2\pi - \frac{\pi}{6} = \frac{12\pi}{6} - \frac{\pi}{6} = \frac{11\pi}{6}$

$\theta = 3x$

$3x_1 = \frac{\pi}{6}$

$x_1 = \frac{\frac{\pi}{6}}{3}$

$x_1 = \frac{\pi}{18}$

$\frac{\frac{\pi}{6}}{\frac{3}{1}} = \frac{\pi}{6} \cdot \frac{1}{3}$

$3x_2 = \frac{11\pi}{6}$

$x_2 = \frac{\frac{11\pi}{6}}{3}$

$x_2 = \frac{11\pi}{18}$

$x_3 = \frac{\pi}{18} + \frac{12\pi}{18} = \frac{13\pi}{18}$

$x_4 = \frac{11\pi}{18} + \frac{12\pi}{18} = \frac{23\pi}{18}$

$\text{Period} = \frac{2\pi}{3} = \frac{12\pi}{18}$

$x_5 = \frac{13\pi}{18} + \frac{12\pi}{18} = \frac{25\pi}{18}$

$x_6 = \frac{23\pi}{18} + \frac{12\pi}{18} = \frac{35\pi}{18}$

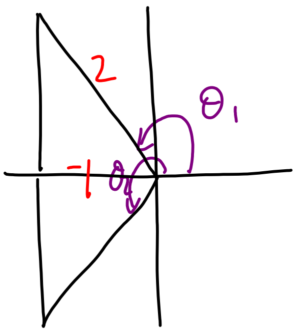
<u>Ex. #3</u>: $\sqrt{2}cos\boxed{2\left(x+\frac{\pi}{3}\right)}+5=4$ Find a general solution using exact values.

$\theta = 2(x+\frac{\pi}{3})$

$\sqrt{2}\cos\theta + 5 = 4$

$\sqrt{2}\cos\theta = -1$

$\cos\theta = \frac{-1}{\sqrt{2}}$



ref $\angle = \frac{\pi}{4}$

$\theta_1 = \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

$\theta_2 = \pi + \frac{\pi}{4}$

$= \frac{5\pi}{4}$

$2(x_1+\frac{\pi}{3}) = \frac{3\pi}{4}$

$x_1 + \frac{\pi}{3} = \frac{3\pi}{8}$

$x_1 + \frac{8\pi}{24} = \frac{9\pi}{24}$

$x_1 = \frac{\pi}{24}$

$2(x_2+\frac{\pi}{3}) = \frac{5\pi}{4}$

$x_2 + \frac{\pi}{3} = \frac{5\pi}{4}$

$x_2 + \frac{8\pi}{24} = \frac{15\pi}{24}$

$x_2 = \frac{7\pi}{24}$

General Solution: Add on (period)n

period $= \frac{2\pi}{2} = \pi$

$x = \frac{\pi}{24} + n\pi$

$x = \frac{7\pi}{24} + n\pi$

$n \in I$

Ex. #4: Solve accurate to 2 decimal places.

$2sin^2x - 3\sin x - 2 = 0 \qquad 0 \le x < 2\pi$

Radian mode

$m = \sin x$

$2m^2 - 3m - 2 = 0$

$\underline{\ \ }\times\underline{\ \ } = -4$

$\underline{-4} + \underline{1} = -3$

$2m^2 - 4m + 1m - 2 = 0$

$2m(m-2)\ 1(m-2) = 0$

$(m-2)(2m+1) = 0$

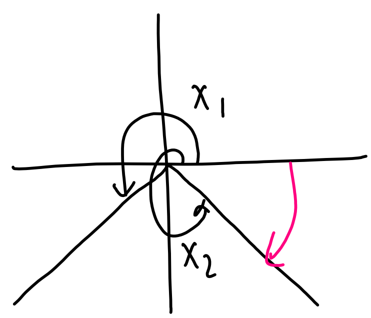
$m = 2 \qquad m = -\frac{1}{2}$

$\sin x = 2$

No solution

$y = \sin x$ has a maximum of 1

$\sin x = -\frac{1}{2}$



$x = \sin^{-1}\left(-\frac{1}{2}\right)$

$x = -.524$

Ref ∠ = .524

$x_1 = \pi + .524$

$x_1 = 3.67$

$x_2 = 2\pi - .524$

$x_2 = 5.76$