5.4 2nd Fundamental Theorem of Calculus

Thursday, June 3, 2021 2:55 PM

AP Calculus

5.4 2nd Fundamental Theorem of Calculus

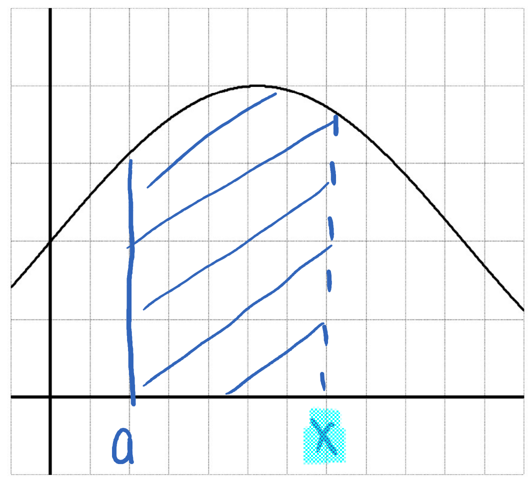
Area function (with lower limit a)

$$A(x) = \int_a^x f(t)dt$$

$$A(a) = \int_a^a f(t)dt$$

$= 0$

No area



1. Find a formula for $A(x) = \int_2^x (t^2 - t)dt$

$$A(x) = \frac{t^3}{3} - \frac{t^2}{2} \Big|_2^x$$

$$A(x) = \frac{x^3}{3} - \frac{x^2}{2} - \left(\frac{(2)^3}{3} - \frac{(2)^{\cancel{2}}}{\cancel{2}}\right)$$

$$A(x) = \frac{x^3}{3} - \frac{x^2}{2} - \frac{8}{3} + 2$$

$$A(x) = \frac{x^3}{3} - \frac{x^2}{2} - \frac{2}{3}$$

$$A'(x) = \frac{\cancel{3}x^2}{\cancel{3}} - \frac{\cancel{2}x}{\cancel{2}} - 0$$

$$A'(x) = x^2 - x$$

(original function)

2nd Fundamental Theorem of Calculus

If $A(x) = \int_a^x f(t)dt$
Then

$$A'(x) = \frac{d}{dt}\int_a^x f(t)\,dt = f(x)$$

2. Find $\frac{d}{dx}\int_1^x (3t^2 + 5t)dt$

$- 3x^2 + 5x$

$$\frac{d}{dx}\left[\frac{\cancel{3}t^3}{\cancel{3}} + \frac{5t^2}{2}\Big|_1^x\right]$$

$$\frac{d}{dx}\left[x^3 + \frac{5}{2}x^2 - \left(1^3 + \frac{5}{2}(1)^2\right)\right]$$

$$= 3x^2 + \frac{5}{\cancel{2}}\cdot\cancel{2}x - 0$$

$$= 3x^2 + 5x$$

3. Find $\frac{d}{dx}\int_5^x \sin t^2\, dt$

$= \sin x^2 \cdot 1$

4. $G(x) = \int_{\frac{\pi}{2}}^{x^3} \cos t \, dt$ Find G'(x)

$G(x) = A(x^3)$

$G'(x) = A'(x^3) \cdot 3x^2$

$G'(x) = \cos(x^3) \cdot 3x^2$

$G'(x) = 3x^2 \cos(x^3)$

Chain rule

2nd Fundamental Theorem of Calculus Using the Chain Rule

If $A(x) = \int_a^u f(t)dt$ where u is a function in terms of x

Then

$A'(x) = \frac{d}{dx} \int_a^u f(t)dt = f(u) \cdot \frac{du}{dx}$

5. Find $\frac{d}{dx} \int_2^{x^2} \frac{1}{t^2} dt$

$= \frac{1}{(x^2)^2} \cdot 2x$

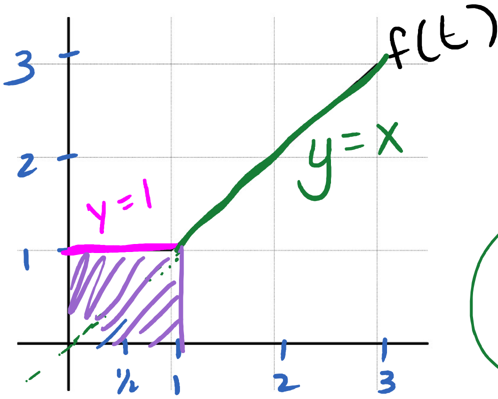
$= \frac{2x}{x^4} = \frac{2}{x^3}$

6. Find $\frac{d}{dx} \int_1^{x^3+2x} \tan t \, dt$

$= \tan(x^3+2x) \cdot (3x^2+2)$

$= (3x^2+2)\tan(x^3+2x)$

7. $A(x) = \int_0^x f(t)dt$



$A(\frac{1}{2}) = \frac{1}{2}$

a) Find a formula for A(x) on [0,1]

input numbers is from 0 to 1

$A'(x) = \frac{d}{dx}\int_0^x f(t)dt$

$A'(x) = f(x)$

on [0,1] $f(x) = 1$

horizontal line

$A(x) = \int_0^x 1\, dt$

$A(x) = t\Big|_0^x$

$A(x) = x - 0$

$A(x) = x$

b) Find a formula for A(x) on [1,3]

Input numbers are from 1 to 3

$A'(x) = \frac{d}{dx}\int_0^x f(t)dt$

$A'(x) = f(x)$

on [1,3] $f(x) = x$

line $y = x$

$A(x) = \int_0^1 f(t)dt + \int_1^x f(t)dt$

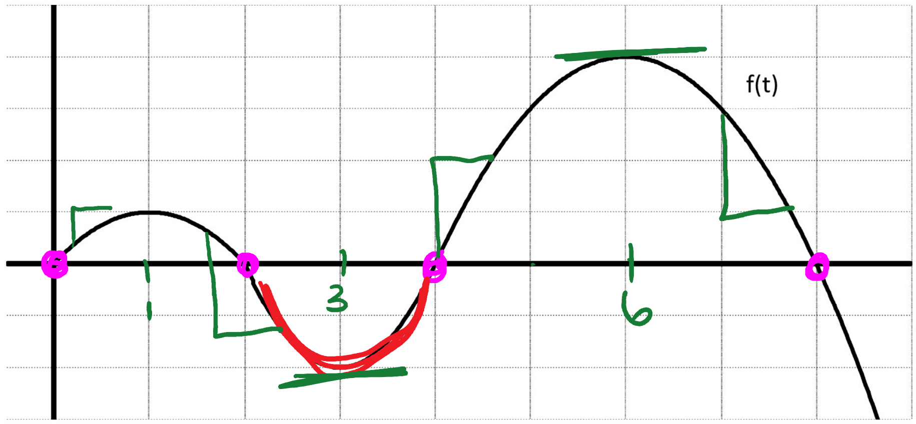
$A(x) = 1 + \int_1^x t\, dt$

$A(x) = 1 + \frac{1}{2}t^2 \Big|_1^x$

$A(x) = 1 + \frac{1}{2}x^2 - \frac{1}{2}(1)^2$

$A(x) = 1 + \frac{1}{2}x^2 - \frac{1}{2} = \frac{1}{2}x^2 + \frac{1}{2}$

8.



$$A(x) = \int_0^x f(t)dt$$

a) Find where any local maximums and minimums of A(x) occur.

Max and Mins occur when $A'(x) = 0$

$A'(x) = \frac{d}{dx}\int_0^x f(t)dt$

* $A'(x) = f(x)$

Find where $A'(x) = 0$

$f(x) = 0$

$x = 0$
$x = 2$
$x = 4$
$x = 8$

(0,2)(2,4)(4,8)(8,9)

sign of $A'(x)$ sign of $f(x)$: + − + −

$x = 2$ Max
$x = 4$ Min
$x = 8$ Max

b) Find where any inflection points of A(x) occur.

$A''(x) = 0$ for inflection points

* $A''(x) = f'(x)$

$A''(x) = 0$

When $f'(x) = 0$

Slope of f(x) is zero

$x = 1$ $x = 3$ $x = 6$

(0,1)(1,3)(3,6)(6,9)

Sign of $A''(x)$ sign of $f'(x)$: + − + −

inflection points at $x = 1$ $x = 3$ and $x = 6$