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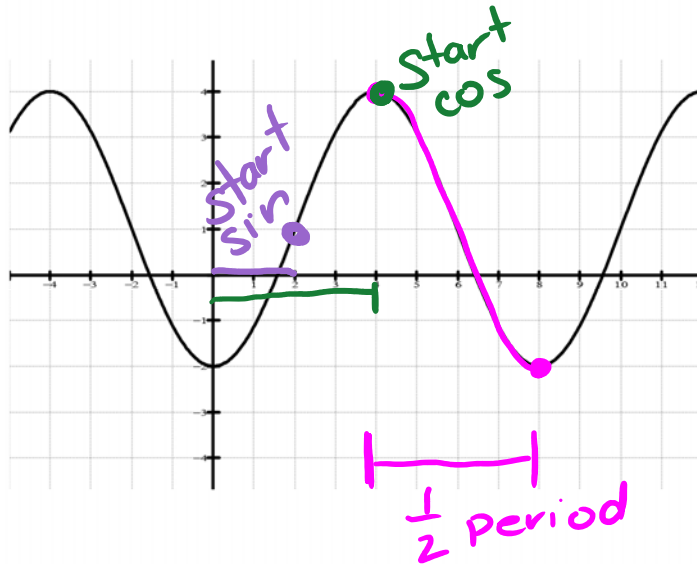
Friday, October 18, 2019

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5.4 Equations and Graphs of Trigonometric Functions: Part II

Ex. #1: The graph below represents a periodic function in the form:

$$y = A \sin B(x - C) + D \quad \text{and} \quad y = A \cos B(x - C) + D$$



a) Find the amplitude and vertical displacement of the function

$$\text{Amp} = \frac{|\text{max} - \text{min}|}{2}$$

$$\text{Amp} = \frac{|4 - (-2)|}{2} = 3$$

$$\text{VD} = \text{Max} - \text{Amp}$$

$$\text{VD} = 4 - 3$$

$$\text{VD} = 1$$

b) Find the period of the function

$$\frac{1}{2} \text{ period} = 8 - 4$$

$$\text{period} = 8$$

$$\text{period} = \frac{2\pi}{b}$$

$$\frac{8}{1} = \frac{2\pi}{b}$$

$$8b = 2\pi$$

$$b = \frac{2\pi}{8}$$

c) Find the phase shift and write the function with respect to $y = \sin x$

Phase shift
right 2 $2 \rightarrow$

$$y = 3 \sin \frac{2\pi}{8} (x - 2) + 1$$

d) Find the phase shift and write the function with respect to $y = \cos x$

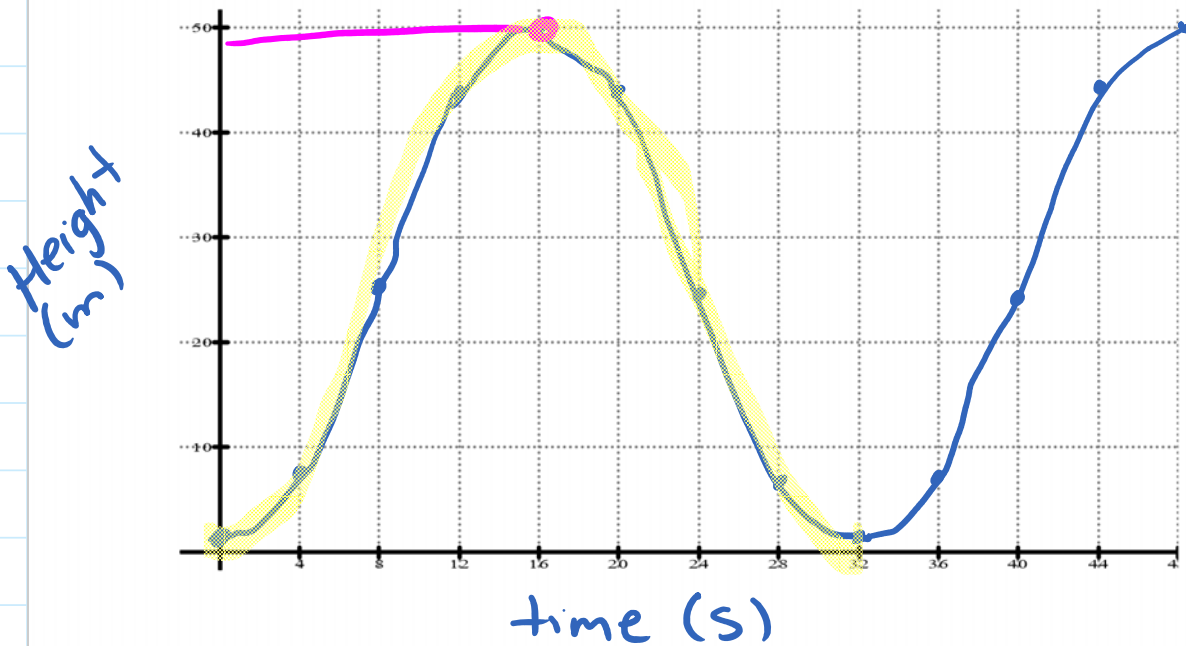
Phase shift
right 4

$$y = 3 \cos \frac{2\pi}{8} (x - 4) + 1$$

Ex. #2: The following data describes the height of a seat on a Ferris wheel above the ground, as the wheel rotates:

Time(s)	0	4	8	12	16	20	24	28	32	36	40	44	48
Height (m)	0.8	8.0	25.4	42.8	50	42.8	25.4	8.0	0.8	8	25.4	42.8	50

(a) Sketch a graph.



(b) What is the amplitude and vertical displacement?

$$\text{Amp} = \frac{|50 - 0.8|}{2} = 24.6 \quad \text{VD} = 50 - 24.6$$

$$\text{VD} = 25.4$$

(c) Determine the phase shift and period with respect to $y = \cos x$.

Phase shift 16 right Period = 32 $b = \frac{2\pi}{32}$

(d) Write the equation in the form $y = A \cos B(x - C) + D$.

$$y = 24.6 \cos \frac{2\pi}{32} (x - 16) + 25.4$$

(e) How high would you be above the ground after 21 seconds?

$$x = 21$$

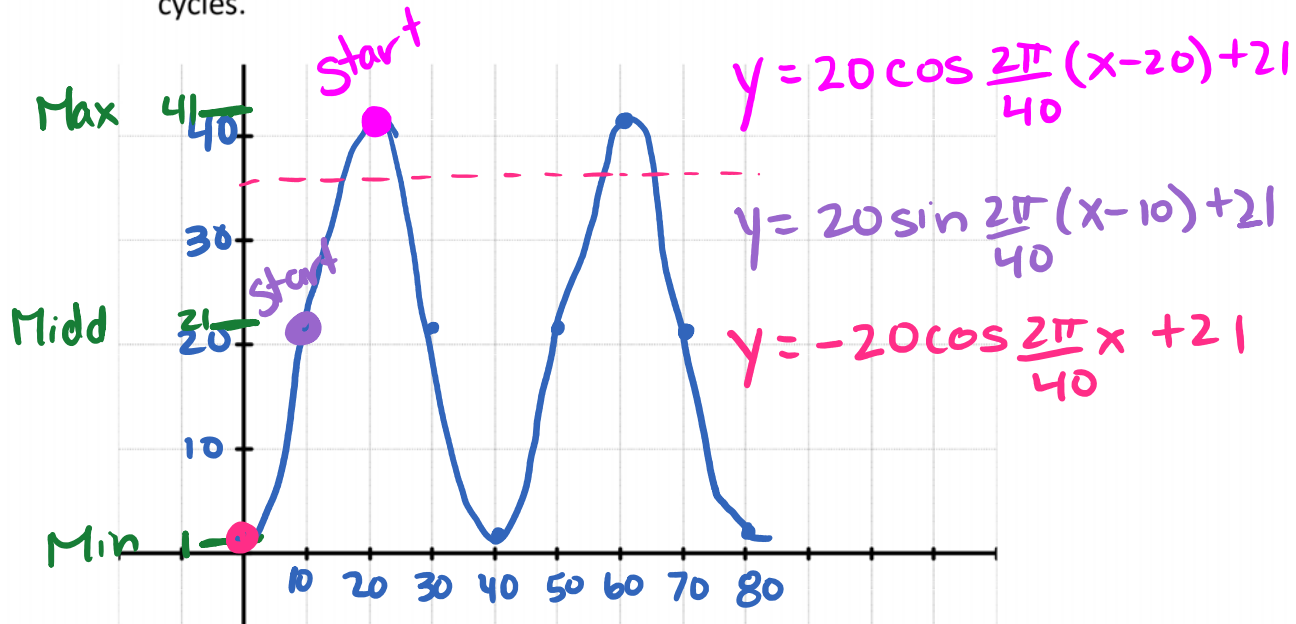
$$y = 24.6 \cos \left(\frac{2\pi}{32} (21 - 16) \right) + 25.4$$

$$y = 39.07 \text{ m}$$

Ex. #2 A ferris wheel has a radius of 20m and rotates once every 40 seconds. Passengers get on 1 m above the ground at point S.

period

(a) Graph how your height above the ground varies during the first 2 cycles.



(b) Write an equation that expresses your height as a function of elapsed time.

Amp = Radius
 Amp = 20
 VD = 41 - 20
 VD = 21

Period = 40
 $b = \frac{2\pi}{40}$

(c) Find your height after 45 seconds.

$x = 45$ $y = 20 \cos \frac{2\pi}{40}(45 - 20) + 21$
 $y = 6.86m$

(d) At what time will you be 35m above the ground? (Make an estimate from your graph)

$x \approx 15, 25, 55$

Ex. #4: The height of a tidal wave above mean sea level is related to time by the function: $h(t) = 1.45 \cos \frac{2\pi(t-2)}{12.4} + 2.35$ where h represents the height, in meters, above mean sea level and t is time, in hours.

(a) What is the maximum height of the wave?

$$\begin{aligned} \text{Max} &= \text{Amp} + VD & \text{Max} &= 3.8 \text{ m} \\ \text{Max} &= 1.45 + 2.35 \end{aligned}$$

(b) In the first cycle, at what time does the maximum occur?

$$t = 2 \text{ hrs}$$

(c) What is the period of the wave?

$$\text{period} = 12.4 \text{ hrs}$$

(d) What is the minimum height of the wave?

$$\begin{aligned} \text{Min} &= -(Amp) + VD & \text{Min} &= 0.9 \text{ m} \\ \text{Min} &= -(1.45) + 2.35 \end{aligned}$$

(e) In the first cycle, at what time does the minimum occur?

$$t = 2 + \frac{12.4}{2} = 8.2 \text{ hrs}$$

↑
max
↑
Half of period

(f) What is the height of the wave 2 hours after high tide?

$$\begin{aligned} &\text{High tide } t = 2 \\ \text{use } &t = 4 \\ &y = 1.45 \cos \frac{2\pi(4-2)}{12.4} + 2.35 \\ &y = 3.12 \text{ m} \end{aligned}$$